ASSESSMENT OF STATIC VAR STABILIZER INPUTS BASED ON ROBUSTNESS CONSIDERATION

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CERTIFICATE

Certified that this work on 'ASSESSMENT OF STATIC VAR STABILIZER INPUTS BASED ON ROBUSTNESS CONSIDERATION' by K. RAJA GOPAL has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

9 September 1991

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TO

MY PARENTS

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ABSTRACT

Conventionally Power System Stabilizers (PSS) are used for the improvement of Dynamic stability of power systems. Dynamic stability can also be improved by using auxiliary controllers in Static Var Systems (SVS).

This thesis considers the problem of designing the most suitable signal for SVS auxiliary controllers for dynamic stability improvement. The input signals considered for these controllers are line power at SVS bus, SVS bus frequency, Computed Internal Frequency (CIF) and a combination of line power and SVS bus frequency. The controller is designed for each case using partial pole placement technique for dynamic output feedback. The power system considered is a single machine - infinite bus system with an SVS at the midpoint of the transmission line and it is modelled on the lines of the Heffron-Philips model.

In the method proposed here, the SVS auxiliary controller is designed at a particular chosen point in the P-Q plane, (P and Q represent respectively the real and reactive power outputs of the generator) and the region in P-Q plane in which the auxiliary controller shows satisfactory performance judged on the basis of a sector criterion for system poles is determined for each input signal. Each region is the region of robustness associated with the corresponding signal. The most efficacious signal for SVS auxiliary controller for dynamic stability improvement is determined by comparing the regions of robustness obtained with each signal.

Using the proposed method the most efficacious signal for SVS auxiliary controller is determined for both weak and strong transmission systems.

SVS bus frequency signal is found to be the most efficacious signal for both weak and strong transmission systems.

LIST OF PRINCIPAL SYMBOLS

Δ	subscript to denote small changes around operating point
o	subscript to denote the value of the operating point
t	time in seconds
ω	actual speed of the generator in radians per second
δ	rotor angle with respect to system reference
δ_m	SVS bus voltage angle
ω_m	SVS bus frequency
δ_c	generator internal voltage angle
ω_c	computed internal frequency
v_t	generator terminal voltage magnitude
V_t	$v_t/\sqrt{3}$
v_m	SVS bus voltage magnitude
V_m	$v_m/\sqrt{3}$
v	infinite bus voltage magnitude
v_{td}, v_{tq}	d and q axes components of v_t
V_{td}, V_{tq}	$v_{td}/\sqrt{3}$, $v_{tq}/\sqrt{3}$
v_{md}, v_{mq}	d and q axes components of v_m
V_{md}, V_{mq}	$v_{md}/\sqrt{3}$, $v_{mq}/\sqrt{3}$
e'_q	speed emf corresponding to the direct axis flux linkage
E'_q	$e'_q/\sqrt{3}$
E_{FD}	d axis stator emf corresponding to the field voltage
E''_d, E''_q	d and q axes components of voltage behind subtransient reactance
i_d, i_q	d and q axes components of armature current
I_d, I_q	$i_d/\sqrt{3}$, $i_q/\sqrt{3}$

i_{1d}, i_{1q}	d and q axes components of SVS current
P	real power output of the generator (p.u.)
Q	reactive power output of the generator (p.u.)
Q_m	SVS reactive power output
R	half line resistance of transmission line in p.u.
X	half line reactance of transmission line in p.u.
x	total line reactance of transmission line in p.u.
B	SVS susceptance
x_t	transformer reactance
x_d, x_q	d and q axes synchronous reactance in p.u.
x'_d	d axis transient reactance in p.u.
x''_d, x''_q	d and q axes subtransient reactance in p.u.
τ'_{do}	d axis open circuit time constant
H	inertia costant in seconds
M	2H
T_m	mechanical torque input to generator (p.u.)
T_e	electrical torque output of generator (p.u.)
K_A	AVR gain
T_A	AVR time constant
K_r	SVS voltage regulator gain
T_r	SVS voltage regulator time constant

Other symbols used in text are explained as and when they are introduced.

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1. INTRODUCTION

1.1 GENERAL

Power system stability is an important factor in planning, design and operation of power systems. Power system stability is usually classified into

(a) Transient stability

(b) Steady state stability

(c) Dynamic stability

Transient stability refers to the stability in the sense of maintenance of synchronism of the system under large disturbances such as a fault on a line, switching of one of several parallel lines out of circuit, combination of a fault and its subsequent isolation by disconnecting part of the system or a sudden and large change in load. Transient stability can be iproved by using fast acting, high gain excitation systems, static var systems, etc.

Steady state stability refers to ability of the system to meet small incremental power demand at an operating point without losing synchronism. Excitation and static var systems help in improving the steady state stability limit.

It is well known that modern fast acting, high gain excitation systems decrease system damping. When damping becomes zero or negative the system becomes unstable. This is referred to as dynamic instability of the system. The instability manifests

itself as low frequency oscillations of sustained or growing nature. These oscillations occur usually in the range of 0.2 to 2.0 Hz. Dynamic instability is more likely to occur when large real powers are transferred, power is delivered at leading power factor or line reactance is large.

Power system stabilizers are widely used to damp out the low frequency oscillations. They are auxiliary controllers located at the generator bus. They commonly receive a feedback signal from either rotor speed or electrical power output and provide a supplementary stabilizing signal to the excitation system of the generator. It has been found that static var systems which are used to improve the voltage profile and reactive power conditions of the system can also damp the low frequency oscillations with a suitably designed auxiliary controller. SVS comprises of a fixed shunt capacitor bank in parallel with a thyristor controlled reactor continuously controlling the inductive current.

PSS and SVS auxiliary controller damp out low frequency oscillations by reactive power control. While PSS modulates reactive power flow through excitation control of generator, SVS auxiliary controller achieves this by controlling SVS reactive power. Various approaches to PSS design are available in the literature. Some of them can be modified and used for SVS auxiliary controller design. A few major works on PSS design methodology are reviewed below.

1.2 REVIEW OF PSS DESIGN METHODOLOGIES

The main approaches to PSS design are design methods based on classical control theory [1,2,3,4], design methods based on minimization of performance index [5,6,7] and design methods based on pole assignment [8,9,10,11,12,13].

deMello and Concordia [1] analysed the problem of dynamic stability in terms of synchronizing and damping torques. Using frequency response technique they designed a phase lead compensator that uses rotor speed deviation as input signal to increase the damping torque.

Larsen and Swann [4] used rotor speed deviation ($\Delta\omega$), rotor frequency deviation (Δf) and change in electrical power of the generator (ΔP) as the input signals to PSS. They compared the performances of these signals with respect to local modes (0.8 to 1.8 Hz) and inter area modes (0.2 to 0.6 Hz). Their study indicates that power input stabilizer provides the best overall local mode damping. The speed input stabilizer provides better local mode damping than the frequency input stabilizer for strong transmission systems whereas frequency input stabilizer provides better local mode damping than speed input stabilizer for weak transmission systems. They suggested that for a good performance over a wide range of contingencies a stabilizer should be designed to provide adequate local mode damping under all operating conditions, with particular attention to condition of heavy load and weak transmission and simulataneously to provide a high contribution to damping of intertia modes.

Anwar [9] used dynamic output feedback methods to shift the eigenvalues corresponding to the rotor mode to suitable locations. The effectiveness of generator power, rotor speed and frequency signals was studied in terms of the stable regions in P-Q plane obtained with these signals. It was reported that the effectiveness of signals varied with system strength. The stable P-Q region obtained for power signal is same for all system strengths. For strong systems frequency signal is found to be better than speed signal whereas for weak systems and higher loading conditions speed signal is found to be better than frequency signal.

Bandhyopadhyay and Prabhu [10] designed a composite adaptive PSS which derives the control signal from the weighted sum of outputs of individual PSS. Madhu [13] designed a Robust adaptive composite PSS using pole placement technique. Angular velocity deviation ($\Delta\omega$), angular acceleration deviation ($\Delta\dot{\omega}$) and both $\Delta\omega$ and $\Delta\dot{\omega}$ (two input PSS) were used as the input signals to PSS. It was found that the performance with a two input PSS is superior to that with a single input PSS.

1.3 IMPROVEMENT OF DYNAMIC STABILITY USING STATIC VAR SYSTEMS : A REVIEW

Reactive power modulation through SVS has been reported to be an effective means of damping the long-term power and voltage oscillations [14]. Schweickardt et al. [15] discussed the necessity of a suitable auxiliary feedback for improving the system damping using SVS.

Kinoshita [16] has studied the effect of SVS on dynamic stability with three stabilizing circuit schemes. The use of power deviation as an auxiliary signal improved the damping of the system significantly. It is also reported by the author that the use of high-gain quick response SVS controller increases the synchronizing power and improves the dynamic stability greatly.

Padiyar and Rajasekharam [17] have used a simple proportional controller as SVS stabilizer with input signals derived from bus voltage and bus frequency deviation. They considered voltage dependent loads and investigated the dynamic stability of the system. They found that frequency feedback improves system damping significantly. Padiyar and Varma [18] have studied the efficacy of various control signals in damping the low frequency oscillations. They compared the signals on the basis of power transfer limits. A new auxiliary control signal, Computed Internal Frequency (CIF) was found to be the best signal. CIF is the synthesized frequency of generator internal voltage from locally available bus voltage, transmission line current and the total inductance between generator internal voltage and local bus. Among the auxiliary signals used they found that line reactive power requires a phase lead compensation in the auxiliary controller whereas bus frequency and CIF require a phase lag compensation in the auxiliary controller.

Ramar and Srinivas [19] designed the SVS stabilizer using pole placement technique. The stabilizer uses line power as the auxiliary signal to damp low frequency oscillations. Hamouda

et al. [20] considered the problem of damping low frequency oscillations as well as torsional modes of steam turbine generators. They used both PSS and SVS for this purpose. They reported that the use of modal speeds as auxiliary signals to SVS result in damping of low frequency oscillations as well as torsional modes.

It is noted that various signals with suitable controllers for SVS have been tried to improve dynamic stability of the system. In actual practice, the operating point, which depends on the load conditions, keeps on varying. The controller designed at a particular operating point may not perform satisfactorily at other operating points. In this thesis, we design an auxiliary controller at a particular operating point (design point) in P-Q plane using pole assignment technique. The performance of the controller as the operating point varies is evaluated by considering the location of the closed loop poles of the system. A sector in the left half of the s-plane is defined based on damping requirements. The operating point of the system is varied and the region in P-Q plane for which the closed loop eigenvalues of the system lie within the sector is found out. This is the region of robustness associated with the design point. The regions of robustness are found out for different signals and the best signal is determined based on these regions for both weak and strong transmission systems.

1.4 SCOPE OF THE PRESENT WORK

A single machine-infinite bus system is considered with an SVS at the mid-point of the transmission line. To damp the low frequency oscillations auxiliary controller for SVS is designed using pole placement technique. Line power, SVS bus frequency, Computed Internal Frequency and a combination of line power and SVS bus frequency are used as input signals to auxiliary controller. The auxiliary controller is designed for each signal at a particular operating point in the P-Q plane. The effectiveness of the signals used is compared in terms of the regions of robustness obtained with each signal and the best signal is determined based on robustness considerations for both weak and strong transmission systems.

A chapterwise summary of the work reported in thesis is as follows.

Chapter 2 deals with the modelling of power system and SVS. A modified Heffron-Philips model represents the system. SVS is represented by a variable susceptance model. State equations for machine, excitation system and SVS with voltage regulator are derived.

In Chapter 3 the design of SVS auxiliary controller is discussed. Partial pole placement technique for dynamic output feedback is applied for the design of auxiliary controller. Expressions for line power, SVS bus frequency and Computed Internal Frequency are derived in terms of state variables and the

application of the pole placement technique with respect to each of these signals is discussed. State equations for the auxiliary controller and the total system are derived. A sector in the s-plane within which the location of eigenvalues is permitted for the analysis of robustness is defined.

Chapter 4 presents the results of the analysis of robustness with the signals considered. Open loop eigenvalues (eigenvalues without SVS), and eigenvalues with only SVS voltage regulator are given. Auxiliary controller parameters and the closed loop eigenvalues at the design point are given for all the signals and system strengths considered. Regions of robustness are plotted for different signals and the best signal is determined on the basis of these regions for both strong and weak systems. Initial condition response for angular position of rotor for different operating points is also presented.

Chapter 5 gives conclusions and suggestions for future work.

2. MODELLING OF THE POWER SYSTEM

2.1 INTRODUCTION

In this study the power system is modelled as a single machine-infinite bus system with an SVS at the mid point of transmission line. The machine-transmission line system is modelled on the lines of the well known model of Heffron-Philips with modifications to include the mid point SVS. A single time-constant model is chosen for excitation system. SVS is modelled as a variable susceptance. The disturbances considered in dynamic stability problem are small and change the state of the system only slightly. Therefore all the system equations are written for small perturbations around operation condition. In this chapter state equations of the system with SVS voltage regulator are derived. The design of auxiliary controller is considered in Chapter 3.

2.2 POWER SYSTEM MODEL

2.2.1 System Under Study

The system under study has a 5000 MVA generator transferring power to an infinite bus over a 500 kV double-circuit transmission line of 200 km length. An SVS is installed at the mid point of the transmission line. The system is same as considered by Kinoshita [16] except for minor modifications. The system is shown in Fig. 2.1. Machine and network constants along with load flow results are given in Appendix 1.

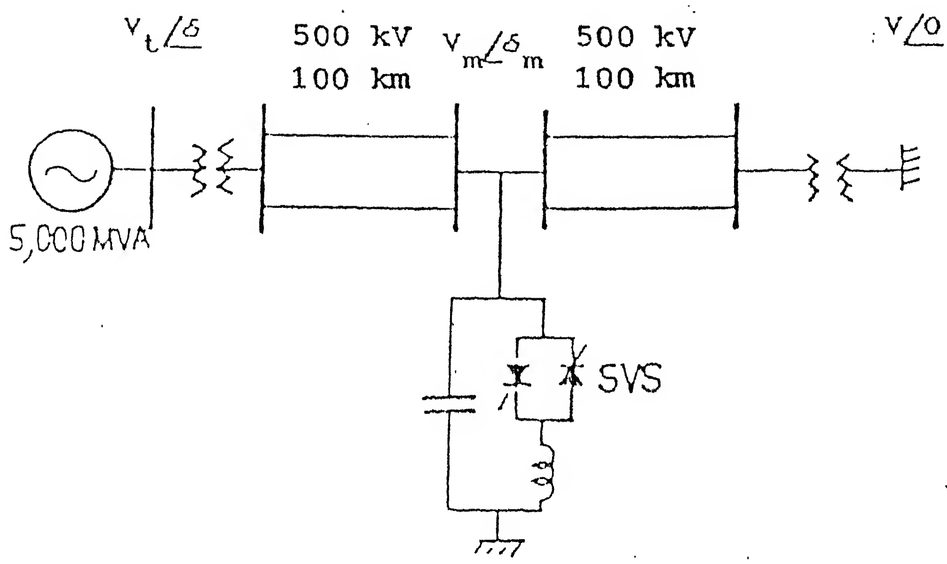


FIG. 2.1 SINGLE MACHINE-INFINITE BUS SYSTEM UNDER STUDY.

2.2.2 Modified Heffron-Philips Model of the System

The system is modelled on the lines of Heffron-Philips model with modifications to include the mid point SVS. The transmission line is represented by a T network and SVS is modelled as a variable susceptance. Transmission line susceptance and SVS susceptance are clubbed together. The simplified model of the system is shown in Fig. 2.2.

2.2.3 Assumptions Made in Developing System Equations

1. Amortisseur effects are neglected.
2. Stator winding resistance is neglected.
3. The voltage terms due to rate of change of flux linkage in time, namely $\dot{\lambda}_d$ and $\dot{\lambda}_q$ are negligible compared to the speed voltage terms $\omega\lambda_q$ and $\omega\lambda_d$.
4. The actual speed ω of the generator is assumed to be close to the nominal speed, ω_R.
5. Balance conditions are assumed and magnetic saturation effects are neglected.

2.2.4 Adequacy of Simplified Linear Model

Since the disturbances considered in dynamic stability problem are small and change the state of the system only slightly it is valid to write the system equations for small perturbations around operating condition.

The damper windings are neglected as their transients die down in a short time after the disturbance to have any effect

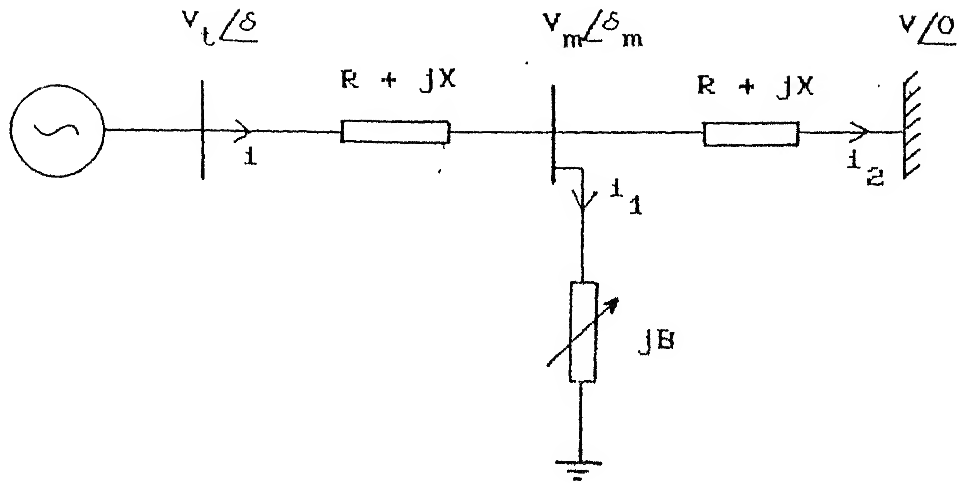


FIG. 2.2 SIMPLIFIED MODEL OF THE SYSTEM.

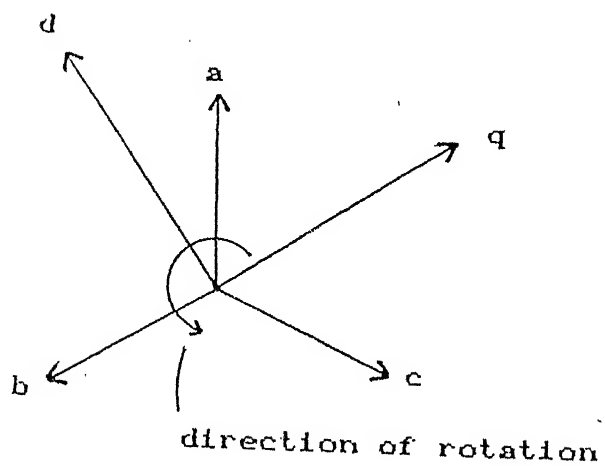


FIG. 2.3 a-b-c AND d-q REFERENCE FRAMES.

on the low frequency oscillations that comprise the dynamic stability problem. The network dynamics are fast compared to the electromechanical dynamics of the rotor of the machine and hence they are neglected and the network is represented by algebraic equations.

Power systems normally operate under balanced conditions and the perturbations considered are assumed not to make the system unbalanced.

2.2.5 Development of the Model

2.2.5.1 Axes of reference

Let $V\angle 0^{\circ}$, $V_m\angle \delta_m$ and $V_t\angle \delta$ represent, respectively, the infinite bus, line mid point and generator terminal voltage phasors (see Fig. 2.2).

The machine d-q axes with respect to the a-b-c winding axes and the direction of rotation are shown in Fig. 2.3.

Using Park's transformation all the system equations are transformed to d-q axes reference frame.

2.2.5.2 Expressions for generator currents

The generator terminal voltage in terms of generator currents and the speed emf corresponding to the direct axis flux linkage, e_q' can be expressed as [22]

$$v_{td} = - x_q \, i_q \qquad (2.1)$$

$$v_{tq} = e'_q + x'_d \, i_d \tag{2.2}$$

The generator terminal voltage can also be written in terms of network currents and infinite bus volotage as (see Fig. 2.2),

$$v_{td} = -\sqrt{3}\, V \sin\delta + R\,(i_d - i_{1d}) + X\,(i_q - i_{1q}) + Ri_d + Xi_q \tag{2.3}$$

$$v_{tq} = \sqrt{3}\, V \cos\delta + R\,(i_q - i_{1q}) - X\,(i_d - i_{1d}) + Ri_q - Xi_d \tag{2.4}$$

Also

$$i_{1d} = (v_{tq} - Ri_q + Xi_d)\, B \tag{2.5}$$

$$i_{1q} = -\,(v_{td} - Ri_d - Xi_q)\, B \tag{2.6}$$

Writing the above equations for small perturbations around operating condition and simplifying we get,

$$\begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \end{bmatrix} \begin{bmatrix} E'_{q\Delta} \\ \delta_\Delta \\ B_\Delta \end{bmatrix} \tag{2.7}$$

where

$$I_{d\Delta} = \frac{i_{d\Delta}}{\sqrt{3}} \qquad I_{q\Delta} = \frac{i_{q\Delta}}{\sqrt{3}} \qquad E'_{q\Delta} = \frac{e'_{q\Delta}}{\sqrt{3}}$$

Expressions for the parameters G_{ij} in eqn. (2.7) are given in Appendix 2.

2.2.5.3 E'_q equation

The d axis stator emf E_{FD} corresponding to the field voltage and E'_q are related by the following equation, [22]

$$E_{FD} = (1 + \tau'_{do}\, s)\; E'_q - (x_d - x'_d)\; I_d \qquad (2.8)$$

Writing eqn. (2.8) for small perturbations around the operating condition and substituting in it for $I_{d\Delta}$ from eqn. (2.7) and rearranging the terms the following equation is obtained

$$E'_{q\Delta} = \left[\frac{K_3}{1+K_3\,\tau'_{do}s}\right] E_{FD\Delta} - \left[\frac{K_3K_4}{1+K_3\,\tau'_{do}s}\right]\delta_\Delta - \left[\frac{K_3K_8}{1+K_3\,\tau'_{do}s}\right] B_\Delta \qquad (2.9)$$

Expressions for the parameters K_3, K_4 and K_8 are given in Appendix 3.

2.2.5.4 Torque equation

The expression for electrical torque of the generator is given by

$$T_e = 3\;[V_d I_d + V_q I_q] \qquad (2.10)$$

Writing eqn. (2.10) for small perturbations around operating point and eliminating $I_{d\Delta}$, $I_{q\Delta}$, $V_{d\Delta}$ and $V_{q\Delta}$ from the resulting equation results in

$$T_e = K_1\delta_\Delta + K_2 E'_{q\Delta} + K_7\, B_\Delta \qquad (2.11)$$

Expressions for the parameters K_1, K_2 and K_7 are given in Appendix 3.

2.2.5.5 Generator terminal voltage equation

The generator terminal voltage in terms of its d-q components can be expressed as

$$V_t^2 = V_d^2 + V_q^2 \tag{2.12}$$

Writing eqn. (2.12) for small perturbations around operating point and simplifying we get,

$$V_{t\Delta} = K_5\delta_\Delta + K_6E'_{q\Delta} + K_9B_\Delta \tag{2.13}$$

Expressions for K_5, K_6 and K_9 are given in Appendix 3.

2.2.5.6 Rotor equations

These equations are given below

$$M\dot{\omega}_\Delta = T_{m\Delta} - T_{e\Delta} \tag{2.14}$$

$$\dot{\delta}_\Delta = \omega_\Delta \tag{2.15}$$

2.2.5.7 State equations for the machine

The dynamics of turbine-speed governer is much slower than that of the generator and therefore in eqn. (2.14) $T_{m\Delta}$ is set to zero. Thus, from eqns. (2.9), (2.11), (2.13), (2.14) and (2.15), we get the state equations of the machine to be

$$\dot{E}'_{q\Delta} = \left[\frac{-1}{K_3\ \tau'_{do}}\right] E'_{q\Delta} - \left[\frac{K_4}{\tau'_{do}}\right]\delta_\Delta + \left[\frac{1}{\tau'_{do}}\right] E_{FD\Delta} - \left[\frac{K_8}{\tau'_{do}}\right] B_\Delta \quad (2.16)$$

$$\dot{\omega}_\Delta = \left(\frac{-K_2}{M}\right) E'_{q\Delta} - \left(\frac{K_1}{M}\right) \delta_\Delta - \left(\frac{K_7}{M}\right) B_\Delta \quad (2.17)$$

$$\dot{\delta}_\Delta = \omega_\Delta \quad (2.18)$$

2.2.5.8 State equation for excitation system

A single time-constant excitation system model was used by many researchers [1,8,9,13,19,22]. This model does not belong to any of the IEEE standard excitation system models [21]. But it is an approximate model suitable for the study of dynamic stability problem. In this study we have choosen the same model to represent the excitation system. A block diagram of the excitation system model is shown in Fig. 2.4.

The state equation for the above system with $V_{ref\Delta} = 0$ is

$$\dot{E}_{FD\Delta} = \left(\frac{-1}{T_A}\right) E_{FD} - \left(\frac{K_A}{T_A}\right) V_{t\Delta}, \quad (2.19)$$

Substituting for $V_{t\Delta}$ from eqn. (2.13)

$$\dot{E}_{FD} = \left(\frac{-1}{T_A}\right) E_{FD\Delta} - \left(\frac{K_A K_5}{T_A}\right) \delta_\Delta - \left(\frac{K_A K_6}{T_A}\right) E'_{q\Delta} - \left(\frac{K_A K_9}{T_A}\right) B_\Delta$$

(2.20)

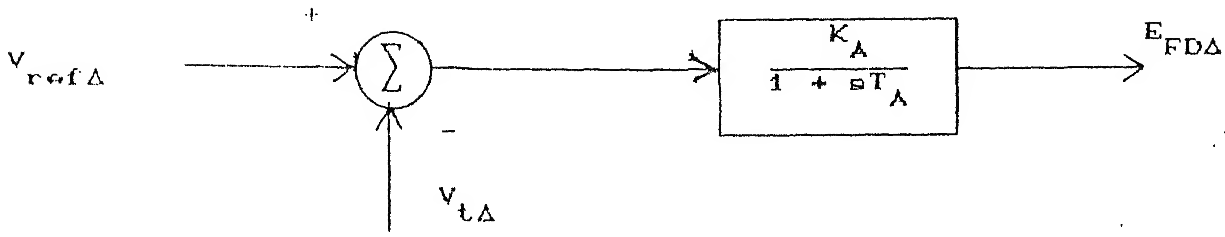


FIG. 2.4 BLOCK DIAGRAM OF EXCITATION SYSTEM.

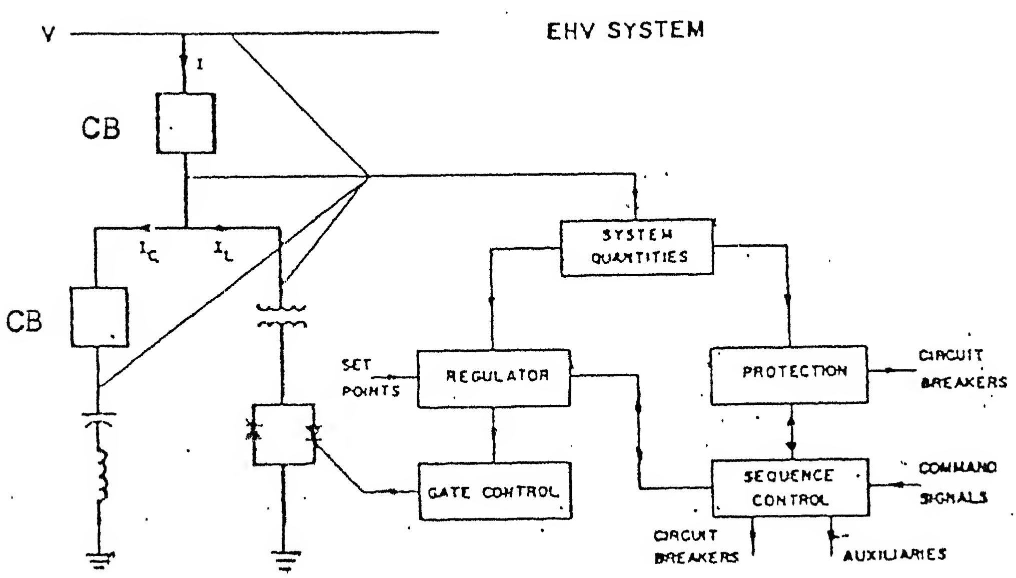


FIG. 2.5 SCHEMATIC DIAGRAM OF FC-TCR.

2.2.5.9 State equations for machine and excitation systems

Eqn.s (2.16), (2.17), (2.18) and (2.20) can be written in matrix form as,

$$\begin{bmatrix} \dot{E}'_{q\Delta} \\ \dot{\omega}_{\Delta} \\ \dot{\delta}_{\Delta} \\ \dot{E}_{FD\Delta} \end{bmatrix} = \begin{bmatrix} \frac{-1}{K_3 \tau'_{do}} & 0 & \frac{-K_4}{\tau'_{do}} & \frac{1}{\tau'_{qo}} \\ \frac{-K_2}{M} & 0 & \frac{-K_1}{M} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-K_A K_6}{T_A} & 0 & \frac{-K_A K_5}{T_A} & \frac{-1}{T_A} \end{bmatrix} \begin{bmatrix} E'_{q\Delta} \\ \omega_{\Delta} \\ \delta_{\Delta} \\ E_{FD\Delta} \end{bmatrix} + \begin{bmatrix} \frac{-K_8}{\tau'_{do}} \\ \frac{-K_7}{M} \\ 0 \\ \frac{-K_A K_9}{T_A} \end{bmatrix} B_{\Delta} \tag{2.21}$$

Eqn. (2.21) is written as

$$\dot{x}_1 = A_1 \underline{x}_1 + \underline{b}_1 u_1 \tag{2.22}$$

where

$$\underline{x}_1 = [E'_{q\Delta} \quad \omega_{\Delta} \quad \delta_{\Delta} \quad E_{FD\Delta}]$$

and A_1 and b_1 are the state and input matrices in eqn. (2.21).

The stability of the system for a particular operating condition and fixed shunt compensation at mid point of transmission line without SVS is studied by finding out the eigenvalues of matrix A_1. It is found that a pair of eigenvalues corresponding to the electromechanical oscillation of the rotor

are located in the right half of the s plane indicating that the system without SVS is unstable (results given in Chapter 4). The effect of SVS on damping is studied by considering a voltage regulator for SVS. The voltage deviation at the mid point of the transmission line is the input signal to SVS voltage regulator.

2.2.5.10 Output equation for machine-excitation system

The voltage deviation at the mid point of the transmission line is the output signal for the machine-excitation system. This signal is fed to the voltage regulator of SVS.

The mid point voltage deviation in terms of $E'_{q\Delta}$, δ_Δ and B_Δ is

$$V_{m\Delta} = [R_1 \quad R_2 \quad R_3] \begin{bmatrix} E'_{q\Delta} \\ \delta_\Delta \\ B_\Delta \end{bmatrix} \tag{2.23}$$

The derivation and expressions for R_1, R_2 and R_3 are given in Appendix 4.

Eqn. (2.23) can be written in the standard form of output equation as,

$$y_1 = C_1 \underline{x}_1 + d_1 u_1 \tag{2.24}$$

where

$$y_1 = V_{m\Delta}$$

$$\underline{x}_1 = [E'_{q\Delta} \quad \omega_\Delta \quad \delta_\Delta \quad E_{FD\Delta}]^t$$

$$C_1 = [R_1 \quad 0 \quad R_2 \quad 0]$$

$$d_1 = R_3$$

Eqns. (2.22) and (2.24) form the first set of state-space equations in this model :

$$\begin{aligned} \dot{\underline{x}}_1 &= A_1\underline{x}_1 + \underline{b}_1 u_1 \\ y_1 &= C_1\underline{x}_1 + d_1 u_1 \end{aligned} \qquad \text{(I)}$$

2.3 MODELLING OF SVS

2.3.1 General

SVS configurations can be broadly classified into two categories :

1. Fixed Capacitor-Thyristor Controlled Reactor (FC-TCR) configuration.
2. Thyristor Switched Capacitor-Thyristor Controlled Reactor (TSC-TCR) configuration.

For small voltage perturbations the performance of both FC-TCR and TSC-TCR is identical [23]. The FC-TCR configuration is considered in this study.

FC-TCR comprises of a thyristor controlled reactor in parallel with a fixed capacitor bank. It is connected to the transmission system through a circuit breaker. It also has the measurement, protection and other auxiliary devices. The schematic diagram of FC-TCR is shown in Fig. 2.5.

The primary function of SVS is to maintain a constant voltage at its terminals through the exchange of reactive power with the transmission system. The steady state control characteristic of SVS is shown in Fig. 2.6.

SVS is preset to maintain its terminal voltage at V_{ref}. For small voltage changes around V_{ref} SVS operates along the segment 1-2 of the control characteristic, [see Fig. 2.6]. Along this segment B_c is kept constant and B_l is varied so that the net susceptance is B_l-B_c. For small voltage rise around V_{ref}, inductive compensation and for small voltage drop around V_{ref}, capacitive compensation are provided.

Large voltage changes take SVS out of its control range. Large overvoltage reduce the action of SVS to that of a shunt reactor with constant susceptance $B_{lmax}-B_c$. SVS acts as a fixed capacitor with constant susceptance $B_{lmin}-B_c$ for large undervoltage. The slope of the segment 1-2 of the control characteristic represents a compromise between SVS rating and the voltage stability requirement.

Fig. 2.7 shows a block diagram of a general control system of SVS.

The three phase voltages at the SVS bus are sensed through PTs by a measuring device. The output of this measuring device is a low voltage dc signal which is compared with reference input V_{ref}. The difference between these two signals is the error signal V_e which is fed to the regulator. Various types of

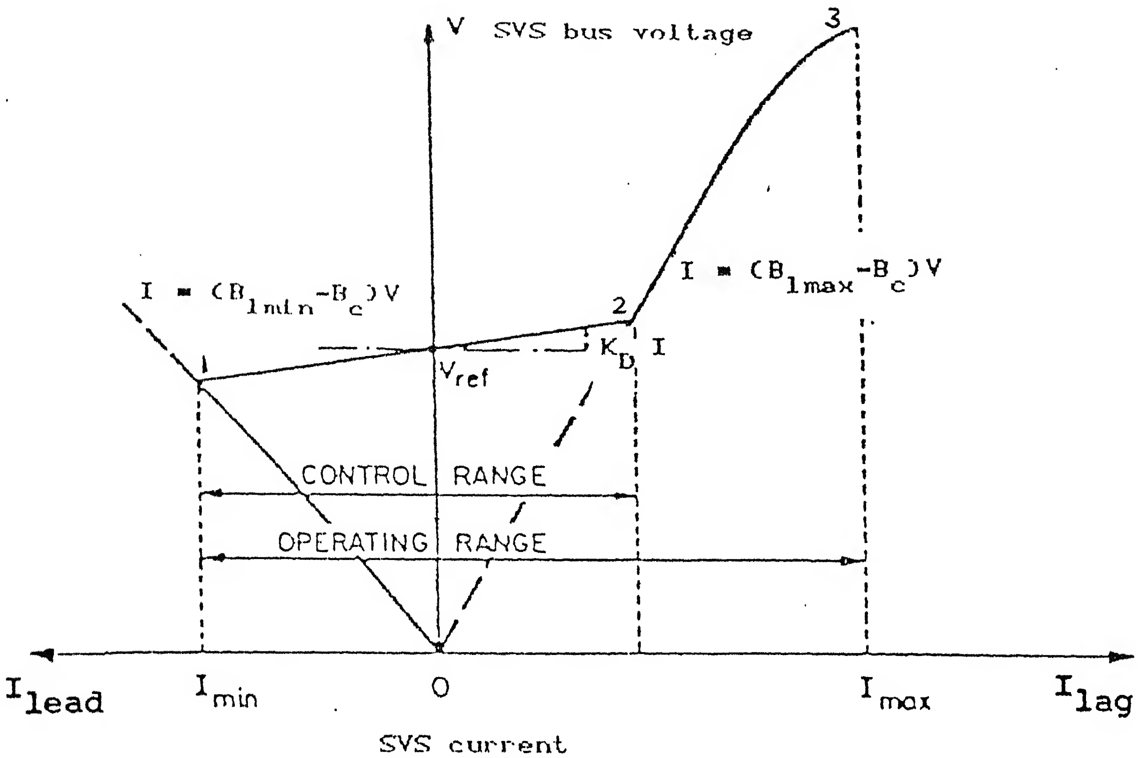


FIG. 2.6 STEADY STATE CONTROL CHARACTERISTIC OF SVS.

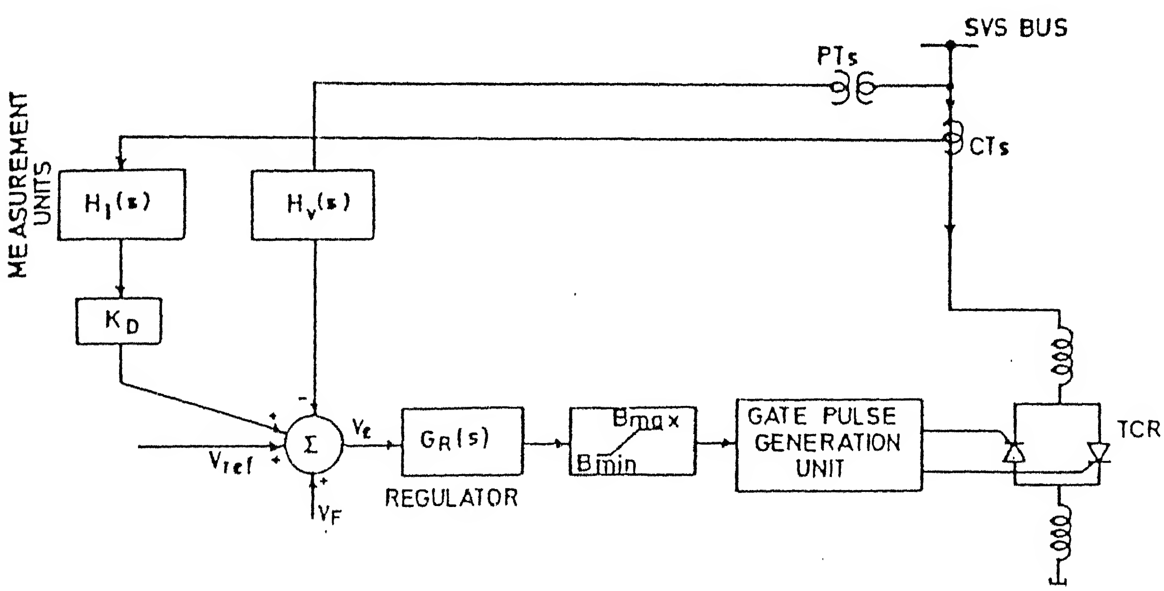


FIG. 2.7 GENERAL CONTROL SYSTEM BLOCK DIAGRAM FOR SVS.

regulators such as simple integral controller [24,25,26], proportional gain with a time constant [16,24,27,28,29], proportional integral controller [15,18], a lead-lag circuit [30] or a lag circuit in cascade with one [31] or two [31] lead-lag transfer functions have been used. The output of the voltage regulator is fed to the gate pulse generation unit which determines the firing angle for TCR. The SVS current input which is added to the summing junction of the regulator through gain K_D results in the desired droop in the V-I characteristic of SVS.

TCR can be represented as a variable susceptance [32] or as an intertialess controllable voltage source behind a fixed reactance [30].

2.3.2 Simplified Model of SVS

In the simplified model, the dynamics of measurement circuits and TCR are neglected. The response times of these circuits are too small, (measurement circuit time constant t $\simeq$ 3 to 5 msec, TCR firing circuit delay $t_s \simeq$ 5 msec and thyristor dead time $t_d \simeq$ 1.667 msec for a six pulse converter), with respect to the low frequency oscillations of the power system in the order of 0.1 to 2.0 Hz, which are to be damped out.

The current droop in the V-I characteristic and the limitations on the size of SVS are ignored as the objective here is to study the damping effects of SVS rather than the voltage regulation of the system. A linear relationship is assumed between the change in SVS susceptance and the change in its firing

angle. A single time constant voltage regulator is considered. A block diagram of the system with SVS controllers is shown in Fig. 2.8.

The performance of SVS without auxiliary signal feedback is studied by developing the system equations without auxiliary controller.

2.3.3 State Equations of the Power System Without SVS Auxiliary Controller

Referring to Fig. (2.9) the state and output equations for SVS voltage regulator are written as

$$\dot{B}_{\Delta} = \left(\frac{-1}{T_r}\right) B_{\Delta} + \left(\frac{K_r}{T_r}\right) (V_{ref\Delta} + y_s - V_{m\Delta}) \qquad (2.24)$$

In the absence of auxiliary controller by setting $V_{ref\Delta}$ to zero, eqn. (2.24) can be written as

$$\dot{B}_{\Delta} = \left(\frac{-1}{T_r}\right) B_{\Delta} + \left(\frac{K_r}{T_r}\right) (- V_{m\Delta}) \qquad (2.24)$$

Eqn. (2.25) is written as

$$\begin{aligned} \dot{x}_2 &= A_2 x_2 + b_2 u_2 \\ y_2 &= C_2 x_2 \end{aligned} \qquad (II)$$

where

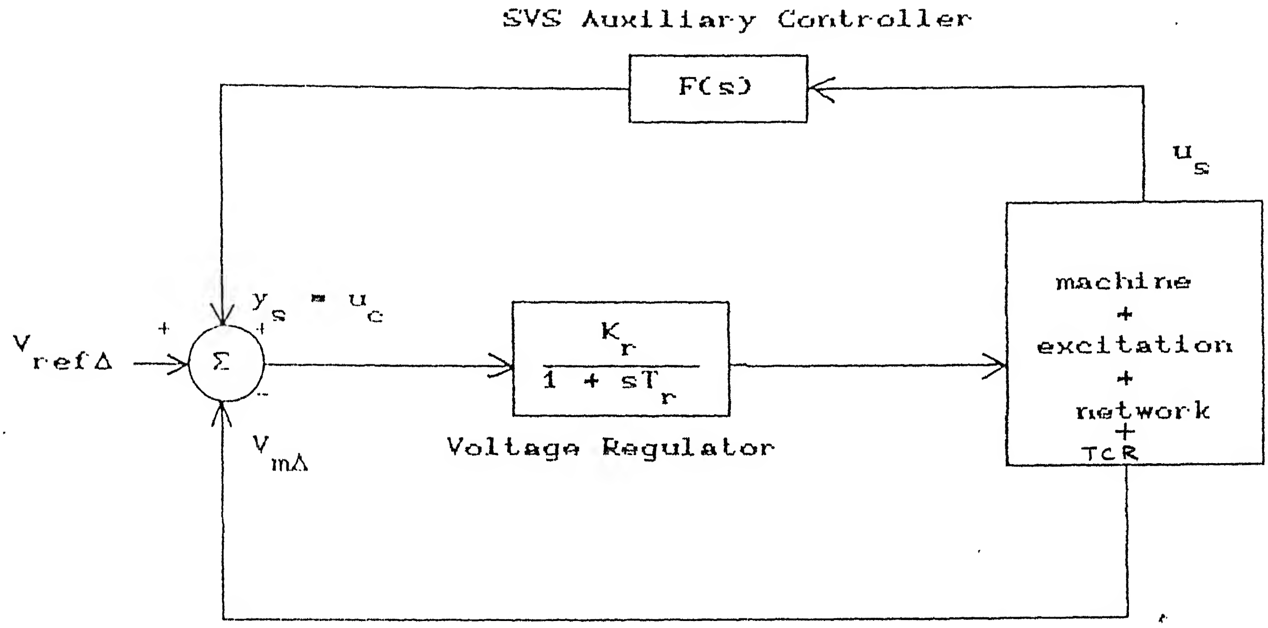


FIG. 2.8 BLOCK DIAGRAM OF THE SYTEM WITH SVS CONTROLLERS.

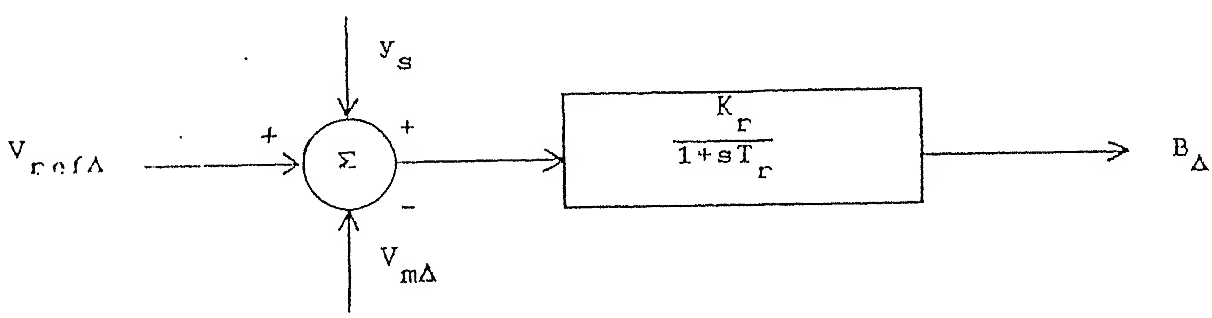


FIG. 2.9 BLOCK DIAGRAM OF SVS VOLTAGE REGULATOR.

$$x_2 = y_2 = B_\Delta \qquad u_2 = -V_{m\Delta}$$

$$A_2 = \frac{-1}{T_r} \qquad b_2 = \frac{K_r}{T_r} \qquad C_2 = 1$$

The machine and excitation system are represented by eqn. (I). Eqn. (II) represents SVS with voltage regulator. Eqns. (I) and (II) can be combined to get

$$\dot{\underline{x}} = A\,\underline{x} \tag{III}$$

where

$$\underline{x} = [E'_{q\Delta} \quad \omega_\Delta \quad \delta_\Delta \quad E_{FD\Delta} \quad B_\Delta]^t$$

Derivation of eqn. (III) is given in Appendix 5. Eqn. (III) represents the state equation of the power system without SVS auxiliary controller.

On finding out the eigenvalues of the system represented by eqn. (III), (results given in Chapter 4), it is found that a pair of eigenvalues corresponding to the electromechanical oscillation of the rotor are located too close to the imaginary axis in s-plane indicating that SVS with only voltage regulator is not capable of damping the low frequency oscillations satisfactorily. Therefore auxiliary signals such as line power, SVS bus frequency, CIF and a combination of line power and SVS bus frequency are used with an auxiliary controller to improve the damping of the system.

3. DESIGN OF SVS AUXILIARY CONTROLLER

3.1 GENERAL

As indicated in the previous chapter, SVS with only voltage regulator is not effective in damping the electromechanical oscillations of the rotor. Therefore an auxiliary controller for SVS is designed in order to provide sufficient damping to the system. When PSS is used to improve the damping of the system it is designed as a phase lead network, [1], and it is found that such a design improves the system damping significantly.

The objective of improving damping of the system can also be viewed as shifting the real parts of the system eigenvalues to more stable locations in s-plane. As it is indicated in the previous chapter, the pair of eigenvalues corresponding to rotor oscillation is found to be critical for dynamic stability of the system. The auxiliary controller is designed to shift these eigenvalues to prespecified locations in the s-plane. The technique of partial pole placement for dynamic output feedbakck as given by Munro and Hirbod [33] is used to design the controller.

3.2 ORDER OF THE COMPENSATOR

A second order compensator is chosen. The number of closed loop poles which can be arbitrarily assigned by output feedback is related to the order r of the compensator as

$$\left[\frac{q-\max(m,l)}{\max\ (m,l)}\right] \leq r \leq q - \max\ (m,l) \qquad (3.1)$$

In eqn. (3.1) [ ] means the integer nearest to the quanity in brackets, q is the number of closed loop poles that can be arbitrarily placed, m is the number of inputs to the system without feedback and l is the number of output of the system without feedback.

In the case of the system considered here m = 1, l = 1 or 2 depending on whether a single input or a two input auxiliary controller is used. With r = 2 and l = 1 we get q = 3 implying that a total number of three closed loop poles can be arbitrarily assigned.

The transfer function of the second order auxiliary controller is

$$F(s) = \frac{t_o s^2 + t_1 s + t_2}{s^2 + g_1 s + g_2} \qquad (3.2)$$

3.3 SECTOR CRITERION

The performance of the auxiliary controller with different signals as the operating point changes is evaluated by considering the location of system closed loop eigenvalues. The mere location of all the closed loop eigenvalues in left half of s-plane does not guarantee satisfactory damping of the system. The nearness of real part of eigenvalues to imaginary axis indicate low damping and for a given real part, large imaginary

part also indicates low damping. Based on this fact a sector in s-plane which restricts the location closed loop eigenvalues is defined as shown in Fig. 3.1. This ensures a minimum damping ratio of 0.068.

3.4 CHOICE OF CLOSED LOOP EIGENVALUES

The following general guidelines are followed in choosing the location of assigned poles :

1. The unassigned poles should be well within the sector.
2. The gains of stabilzing function should not be too high.
3. The robustness regions obtained based on sector criterion should be satisfactory.

3.5 STATE EQUATIONS OF THE SYSTEM WITH AUXILIARY CONTROLLER

A block diagram of the system with SVS controllers is shown in Fig. 2.8.

The state equations of system without auxiliary controller is given by eqn. (III). Eqn. (III) can be modified as below to include the input u_c from the auxiliary controller

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}_c\, u_c \qquad \text{(IV)}$$

where

$$\underline{x} = [E'_{q\Delta} \quad \omega_\Delta \quad \delta_\Delta \quad E_{FD\Delta} \quad B_\Delta]^t$$

$$b_c = [\,0 \quad 0 \quad 0 \quad 0 \quad b_2\,]^t$$

and matrix A is same as in eqn. (III).

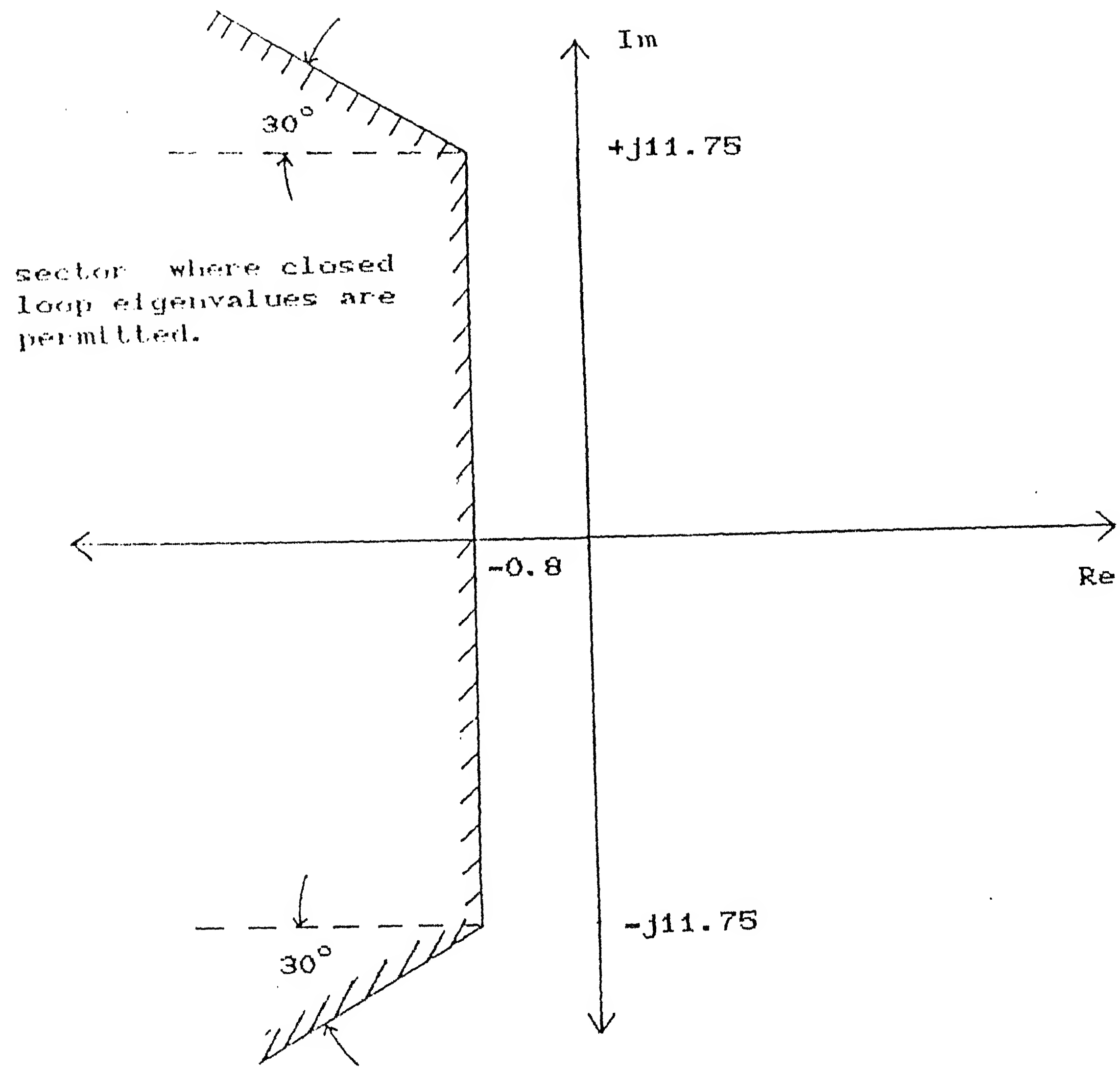


FIG. 3.1 SECTOR INDICATING PERMISSIBLE LOCATION OF CLOSED LOOP EIGENVALUES.

3.6 EXPRESSIONS FOR SIGNALS USED

Line power at SVS bus, SVS bus frequency, Computed Internal Frequency and a combination of line power and SVS bus frequency are the input signals used with auxiliary controller. The expressions for these signals in terms of state variables are given below.

3.6.1 Expression for Line Power at SVS Bus

Referring to Fig. 2.2 the line power at SVS bus can be written as

$$P_m = v_{md}\, i_d + v_{mq}\, i_q \tag{3.3}$$

Writing the above equation for small perturbations around operating condition and simplifying we get,

$$P_{m\Delta} = \omega_1\, E'_{q\Delta} + \omega_2\, \delta_\Delta + \omega_3\, B_\Delta \tag{3.4}$$

Derivations of expressions for ω_1, ω_2 and ω_3 are given in Appendix 4.

Eqn. (3.4) in terms of state variables is

$$y_{c1} = C_{c1}\, \underline{x} \tag{3.5}$$

where

$$y_{c1} = P_{m\Delta}$$

$$\underline{x} = [E'_{q\Delta} \quad \omega_\Delta \quad \delta_\Delta \quad E_{FD\Delta} \quad B_\Delta]^t$$

$$C_{c1} = [\omega_1 \quad 0 \quad \omega_2 \quad 0 \quad \omega_3]$$

3.6.2 Expression for SVS Bus Frequency

The expression for SVS bus voltage angle is

$$\delta_m = \tan^{-1}\left[\frac{V_{md}}{V_{mq}}\right] \tag{3.6}$$

SVS bus frequency can be written as

$$\omega_m = \frac{d}{dt}\,\delta_m \tag{3.7}$$

Substituting eqn. (3.6) in eqn. (3.7) we get

$$\omega_m = \frac{d}{dt}\tan^{-1}\left[\frac{V_{md}}{V_{mq}}\right] \tag{3.8}$$

Writing eqn. (3.8) for small perturbations around operating point and simplifying we get

$$y_{c2} = C_{c2}\,\underline{x} + d_{c2}\,u_c \tag{3.9}$$

where $y_{c2} = \omega_{m\Delta}$

$$\underline{x} = [E'_{q\Delta} \quad \omega_{\Delta} \quad \delta_{\Delta} \quad E_{FD\Delta} \quad B_{\Delta}]^t$$

u_c is input from auxiliary controller, C_{c2} and d_{c2} are the output matrices.

Derivations of the expressions for C_{c2} and d_{c2} are given in Appendix 4.

3.6.3 Expression for Computed Internal Frequency

Computed Internal Frequency is the synthesized frequency of generator internal voltage from locally available bus voltage, transmission line current and the total inductance between generator internal voltage and the local bus, [18].

An expression for CIF is derived using the generator voltage behind subtransient reactance, line current and SVS bus voltage as was done by Varma et al. [18].

The generator terminal voltage in terms of voltage behind subtransient reactance is, [22],

$$V_{td} = - I_q x'' + E''_d \tag{3.10}$$

$$V_{tq} = - I_d x'' + E''_q \tag{3.11}$$

where

$$x'' = x''_d = x''_q$$

The generator terminal voltage in terms of SVS bus voltage and line current is

$$V_{td} = V_{md} + RI_d + XI_q \tag{3.12}$$

$$V_{tq} = V_{mq} + RI_q - XI_d \tag{3.13}$$

From eqns. (3.10), (3.11), (3.12) and (3.13) we get

$$E''_d = V_{md} + RI_d + (X + x'')\, I_q \tag{3.14}$$

$$E''_q = V_{mq} - (X + x'')\ I_d + RI_q \tag{3.15}$$

The internal voltage angle δ_c is

$$\delta_c = \tan^{-1}\left[\frac{E''_d}{E''_q}\right] \tag{3.16}$$

Computed internal frequency is

$$\omega_c = \frac{d}{dt}\ \delta_c \tag{3.17}$$

Substituting eqn. (3.16) in eqn. (3.17) we get

$$\omega_c = \frac{d}{dt}\ \tan^{-1}\left[\frac{E''_d}{E''_q}\right] \tag{3.18}$$

Eqns. (3.14), (3.15) and (3.18) can be simplified and written for small perturbations around operating point in terms of state variables as

$$y_{c3} = C_{c3}\ \underline{x} + d_{c3}\ u_c \tag{3.19}$$

where $y_{c3} = \omega_{c\Delta}$

$$\underline{x} = [E'_{q\Delta}\ \ \omega_{\Delta}\ \ \delta_{\Delta}\ \ E_{FD\Delta}\ \ B_{\Delta}]^t$$

u_c is the input from auxiliary controller C_{c3} and d_{c3} are output matrices.

Derivations of expressions for C_{c3} and d_{c3} are given in Appendix 4.

Eqn. (IV) along with eqns. (3.5), (3.9) and (3.19) are the system equations with different signals. These equations are summarized below.

System equations with line power signal

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}_c\,u_c$$

$$y_{c1} = C_{c1}\,\underline{x} \qquad \text{(IV.a)}$$

System equations with SVS bus frequency signal

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}_c\,u_c$$

$$y_{c2} = C_{c2}\,\underline{x} + d_{c2}\,u_c \qquad \text{(IV.b)}$$

System equations with CIF signal

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}_c\,u_c$$

$$y_{c3} = C_{c3}\,\underline{x} + d_{c3}\,u_c \qquad \text{(IV.c)}$$

System equations with line power and SVS bus frequency :

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}_c\,u_c$$

$$y_{c1} = C_{c1}\,\underline{x}$$

$$y_{c2} = C_{c2}\,\underline{x} + d_{c2}\,u_c \qquad \text{(IV.d)}$$

3.7 POLE PLACEMENT ALGORITHM

The algorithm of partial pole placement technique for dynamic output feedback is used to stabilize the systems represented by eqns. (IV.a), (IV.b), (IV.c) and (IV.d) with the stabilizer of the form given by eqn. (3.2). The algorithm is given in Appendix 6.

In the case of line power signal, the algorithm given by Munro and Hirbod is directly applied whereas in the case of SVS bus frequency and CIF signals, the systems to be stablized have a feed forward term due to the coefficients d_{c2} and d_{c3} not being zero and the algorithm of Munro and Hirbod is not directly applicable. It has to be modified. In the modified method only two poles are shifted to prespecified location and one of the stabilizer parameters, t_o in this case, is also assigned a value. The unassigned poles and the remaining controller parameters are obtained in a way similar to that in the original algorithm. The procedure is repeated for different values of t_o till a satisfactory solution in terms of unassigned poles is obtained.

3.8 STATE EQUATIONS OF THE AUXILIARY CONTROLLER

A block diagram of the auxiliary controller is shown in Fig. 3.2.

We can derive the following state and output equations for the controller

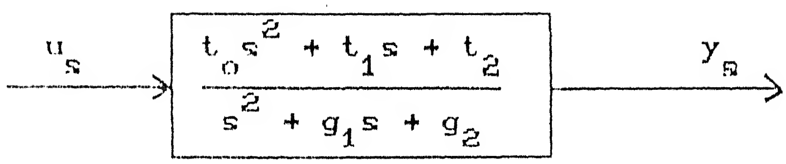


FIG. 3.2 BLOCK DIAGRAM OF SVS AUXILIARY CONTROLLER.

$$\begin{bmatrix} \dot{x}_{s1} \\ \dot{x}_{s2} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -g_2 & -g_1 \end{bmatrix} \begin{bmatrix} x_{s1} \\ x_{s2} \end{bmatrix} + \begin{bmatrix} 0 \\ t_o \end{bmatrix} u_s \tag{3.20}$$

$$y_s = \begin{bmatrix} \frac{t_2}{t_o} - g_2 & \frac{t_1}{t_o} - g_1 \end{bmatrix} \begin{bmatrix} x_{s1} \\ x_{s2} \end{bmatrix} + t_o\, u_s \tag{3.21}$$

Derivation of the eqns. (3.20) and (3.21) is given in Appendix 7. Eqns. (3.20) and (3.21) are written as

$$\dot{\underline{x}}_s = A_s\, \underline{x}_s + \underline{b}_s\, u_s$$

$$y_s = C_s\, \underline{x}_s + d_s\, u_s \tag{V}$$

3.9 CLOSED LOOP SYSTEM WITH SVS AUXILIARY CONTROLLER

The closed loop system with SVS auxiliary controller for line power, SVS bus frequency, CIF, combination of line power and SVS bus frequency is obtained by combining eqns. (IV.a), (IV.b), (IV.c) and (IV.d) respectively with eqn. (V). The general form of the resulting equations is

$$\dot{\underline{x}}_f = A_f\, \underline{x}_f$$

where

$$\underline{x}_f = [E'_{q\Delta} \quad \omega_\Delta \quad \delta_\Delta \quad E_{FD\Delta} \quad x_2 \quad x_{s1} \quad x_{s2}]^t$$

The closed loop eigenvalues are determined at different operating points for each of the signals for both weak and strong systems and based on the sector criterion the regions of robustness are found out. Results are presented in Chapter 4.

4. PERFORMANCE OF SVS WITH VARIOUS INPUT SIGNALS

4.1 GENERAL

In Chapter 2 it is pointed out that SVS with only voltage regulator is not effective in damping the electromechanical oscillations of rotor. An auxiliary controller for SVS is designed based on the design methodology presented in the previous chapter.

Line power, SVS bus frequency, CIF and a combination of line power and SVS bus frequency are the input signals to the auxiliary controller. The controller is designed at full load, unity power factor condition for each of the signals for both weak and strong systems. In the weak system the generator is connected to the infinite bus by a single double circuit line with total transmission system inductive reactance being 0.652 p.u. on a 500 kV, 5000 MVA base ($x = 0.652$). In the strong system, the generator is connected to the infinite bus by two double circuit lines with total transmission system inductive reactance being 0.426 p.u. ($x = 0.426$) on the base chosen. The regions of robustness are found out for all the signals based on sector criterion. The most efficacious signal is found out for both weak and strong systems. The numerical results are presented below.

4.2 PERFORMANCE WITH ONLY SVS VOLTAGE REGULATOR

Table 4.1 shows the open loop eigenvalues for different system strengths.

Table 4.1 Open loop eigenvalues for weak and strong systems.

Weak system (x = 0.652)

Open loop eigenvalues	
- 94.3991147513	
- 10.0928686946	
2.0681638738	+j9.6614207512
2.0681638738	-j9.6614207512

Strong system (x = 0.426)

Open loop eigenvalues	
- 95.1789471385	
- 7.5137038583	
1.1335578458	+j11.1740741624
1.1335578458	-j11.1740741624

Table 4.2 Eigenvalues with only SVS voltage regulator for weak and strong systems.

Weak system (x = 0.652)

Eigenvalues with only SVS voltage regulator	
-134.395228	
- .333149	+j10.614631
- .333149	-j10.614631
- 2.543204	
-102.319143	

Strong system (x = 0.426)

Eigenvalues with only SVS voltage regulator	
- 96.967804	+j12.967576
- 96.967804	-j12.967576
- .378807	+j11.668060
- .378807	-j11.668060
- 2.358224	

The open loop eigenvalues clearly indicate that the rotor mode is unstable. Therefore SVS with voltage regulator is considered to stabilize the system. The gain and time constants of the regulator are chosen to be K_s = 20.0 and t_s = 0.05 sec.

System eigenvalues with only SVS voltage regulator are given in Table 4.2.

It is evident from the eigenvalues with SVS voltage regulator that the system has become stable but the rotor oscillations are poorly damped as indicated by the real part of the corresponding eigenvalues being close to imaginary axis.

4.3 PERFORMANCE WITH AUXILIARY CONTROLLER

Auxiliary controller is designed with its poles at -15.0 and -10.0. Higher values such as -20.0 and -25.0 are also tried but there is no specific advantage with these values. Out of the five eigenvalues obtained with only SVS voltage regulator, three can be shifted to prespecified locations in case of power signal whereas only two can be shifted in the case of other signals. The real parts of the complex conjugate pair, which represent the dominant mode of electromechanical oscillations are shifted to -1.5 and the imaginary parts are not changed from their original values for all the signals considered. The locations of the closed loop eigenvalues are selected by a trial-and-error process based on the guidelines given in Section 4.

Table 4.3 Numerical results with line power signal for a weak system.

Input signal to SVS auxiliary controller : line power

System strength : x = 0.652

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

```
-134.395228
-    .333149     +j10.614631
-    .333149     -j10.614631
-   2.543204
-102.319143
```

Assigned poles

```
-1.5+/-j10.6099
-2.54
```

Auxiliary controller transfer function

$$F(s) = \frac{-8.011279962111750E-02\ s^{2} - 4.223215090867482E-03\ s + 3.351048586477511E-05}{1.0\ s^{2} + 7.95800E-02\ s + 1.5200E-03}$$

Closed loop eigenvalues

```
-110.606177327
- 85.413133954
-   1.499983375    +j10.609957747
-   1.499983375    -j10.609957747
-   2.540018364
-  18.025281489
-  11.515659283
```

Range of robustness on 1.0 p.u. MVA semicircle

```
(.83,  0.557763)
         to
(.95, -0.312250)
```

Table 4.4 Numerical results with line power signal for a strong system.

Input signal to SVS auxiliary controller : line power

System strength : x = 0.426

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

- 96.967804	+j12.967576
- 96.967804	-j12.967576
- .378807	+j11.668060
- .378807	-j11.668060
- 2.358224	

Assigned poles

-1.5+/-j11.6599
-2.36

Auxiliary controller transfer function

$$F(s) = \frac{-4.1373704E{-}02\ s^2 - 3.47778214E{-}03\ s + 4.48967783{-}06}{1.0\ s^2 + 7.95800E{-}02\ s + 1.5200E{-}03}$$

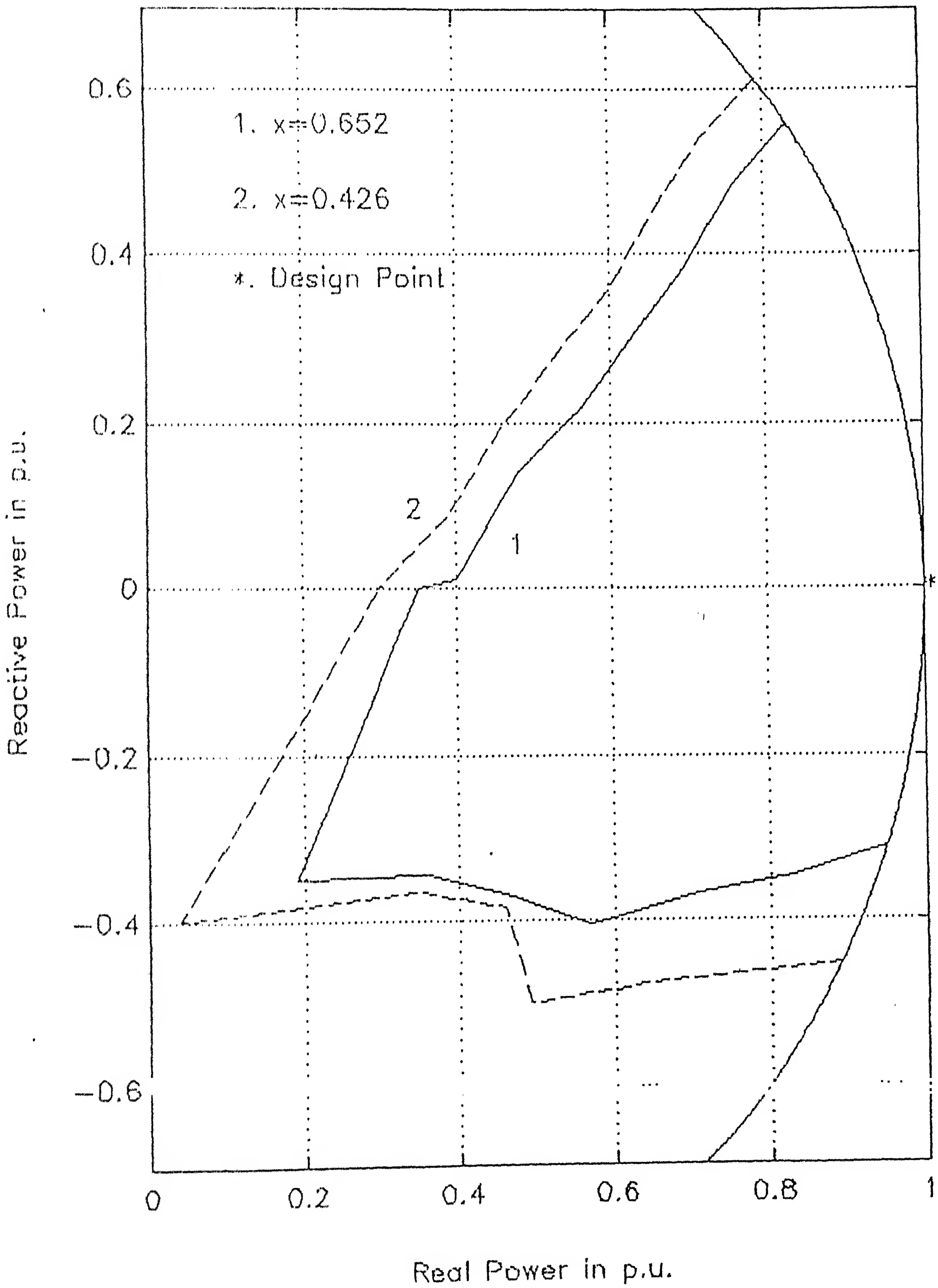
Closed loop eigenvalues

- 93.094772399	
- 91.201964451	
- 1.500000000	+j11.659996740
- 1.500000000	-j11.659996740
- 11.858929405	+j 5.045153975
- 11.858929405	-j 5.045153975
- 2.360000000	

Range of robustness on 1.0 p.u. MVA semicircle

(.79, 0.613107)
to
(.89, -0.455961)

Fig.4.1Regions of Robustness with SVS Bus Power (Weak & Strong systems)



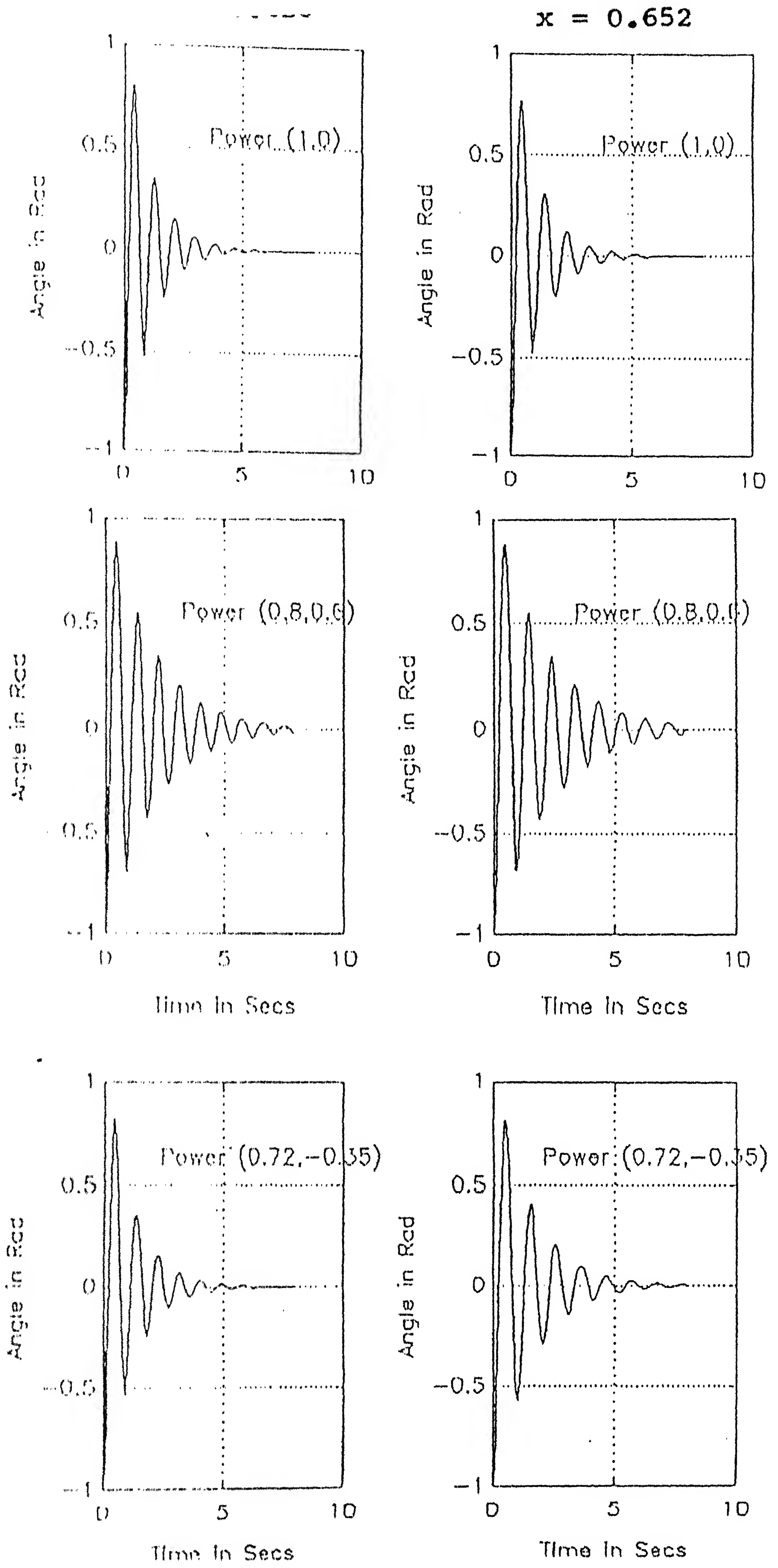


FIG. 4.2 INITIAL CONDITION RESPONSE WITH POWER SIGNAL.

Table 4.5 Numerical results with SVS bus frequency signal for a weak system.

Input signal to SVS auxiliary controller : SVS bus frequency

System strength : x = 0.652

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

-134.395228	
- .333149	+j10.614631
- .333149	-j10.614631
- 2.543204	
-102.318143	

Assigned poles

-1.5+/-j10.61

Auxiliary controller transfer function

$$F(s) = \frac{9.99999E-03\ s^2 + .295206901753591\ s + 1.154687328547005E-02}{1.0\ s^2 + 7.95800E-02\ s + 1.52000E-03}$$

Closed loop eigenvalues

-111.256793402	+j14.595213741
-111.256793402	-j14.595213741
- 1.500000000	-j10.609982506
- 1.500000000	+j10.609982506
- 5.402499130	+j 0.635023991
- 5.402499130	-j 0.635023991
- 13.851970867	

Range of robustness on 1.0 p.u. MVA semicircle

(.84, 0.542586)
to
(.91, -0.414608)

Table 4.6 Numerical results with SVS bus frequency signal for a strong system.

Input signal to SVS auxiliary controller : SVS bus frequency

System strength : x = 0.426

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

- 96.967804	+j12.967576
- 96.967804	-j12.967576
- .378807	+j11.668060
- .378807	-j11.668060
- 2.358224	

Assigned poles

-1.5+/-j11.668058

Auxiliary controller transfer function

$$F(s) = \frac{9.99999E-03\ s^2 + .2790635468\ s + 1.2278098949E-02}{1.0\ s^2 + 7.95800E-02\ s + 1.5200E-03}$$

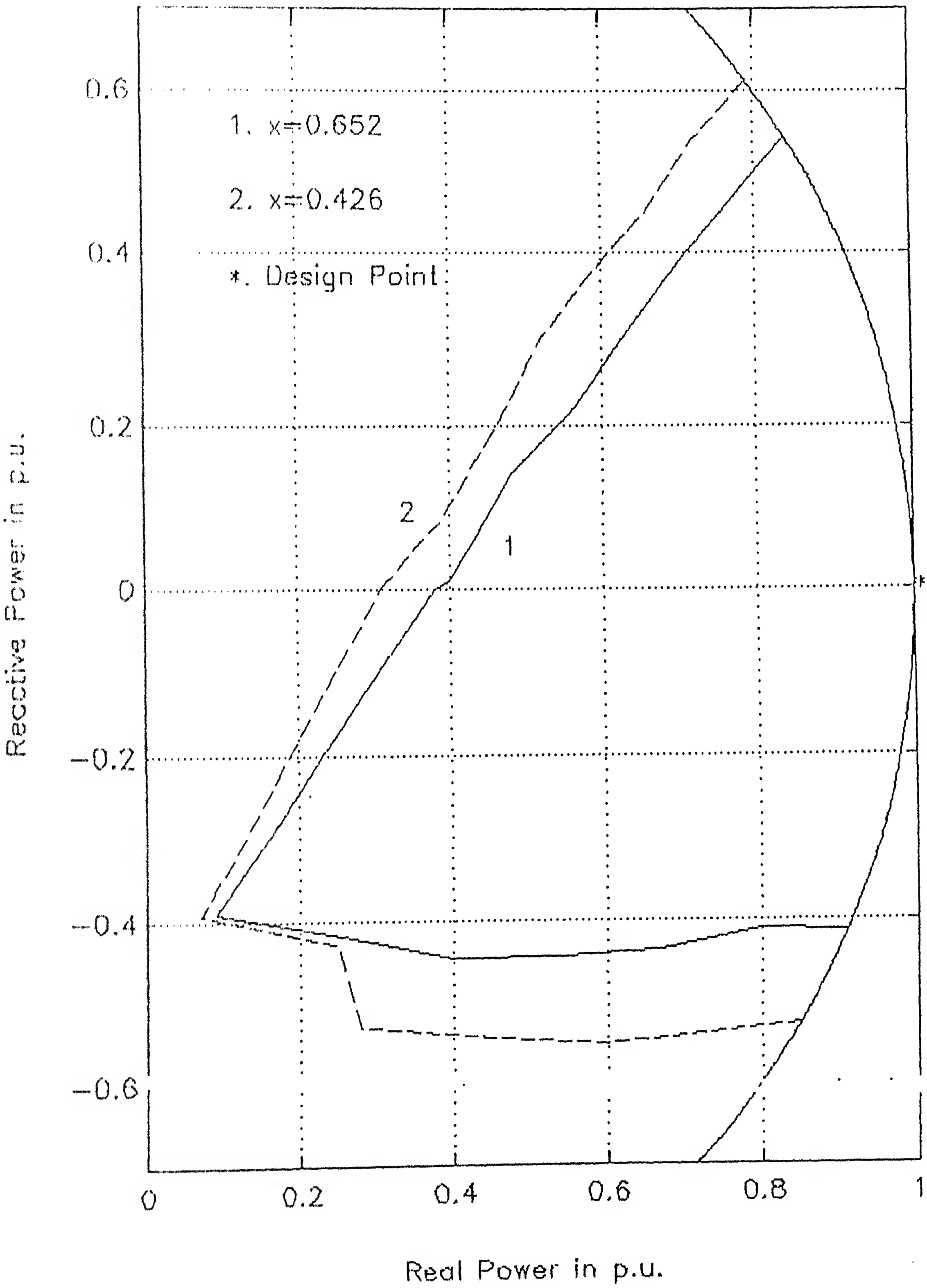
Closed loop eigenvalues

- 94.143056214	+j16.044848156
- 94.143056214	-j16.044848156
- 1.500000000	+j11.599977837
- 1.500000000	-j11.599977837
- 4.934437865	+j 1.013555045
- 4.934437865	-j 1.013555045
- 14.583818663	

Range of robustness on 1.0 p.u. MVA semicircle

(.79, 0.613107)
to
(.85, -0.526783)

Fig.4.3Regions of Robustness with SVS Bus Freq. (Weak & Strong systems)



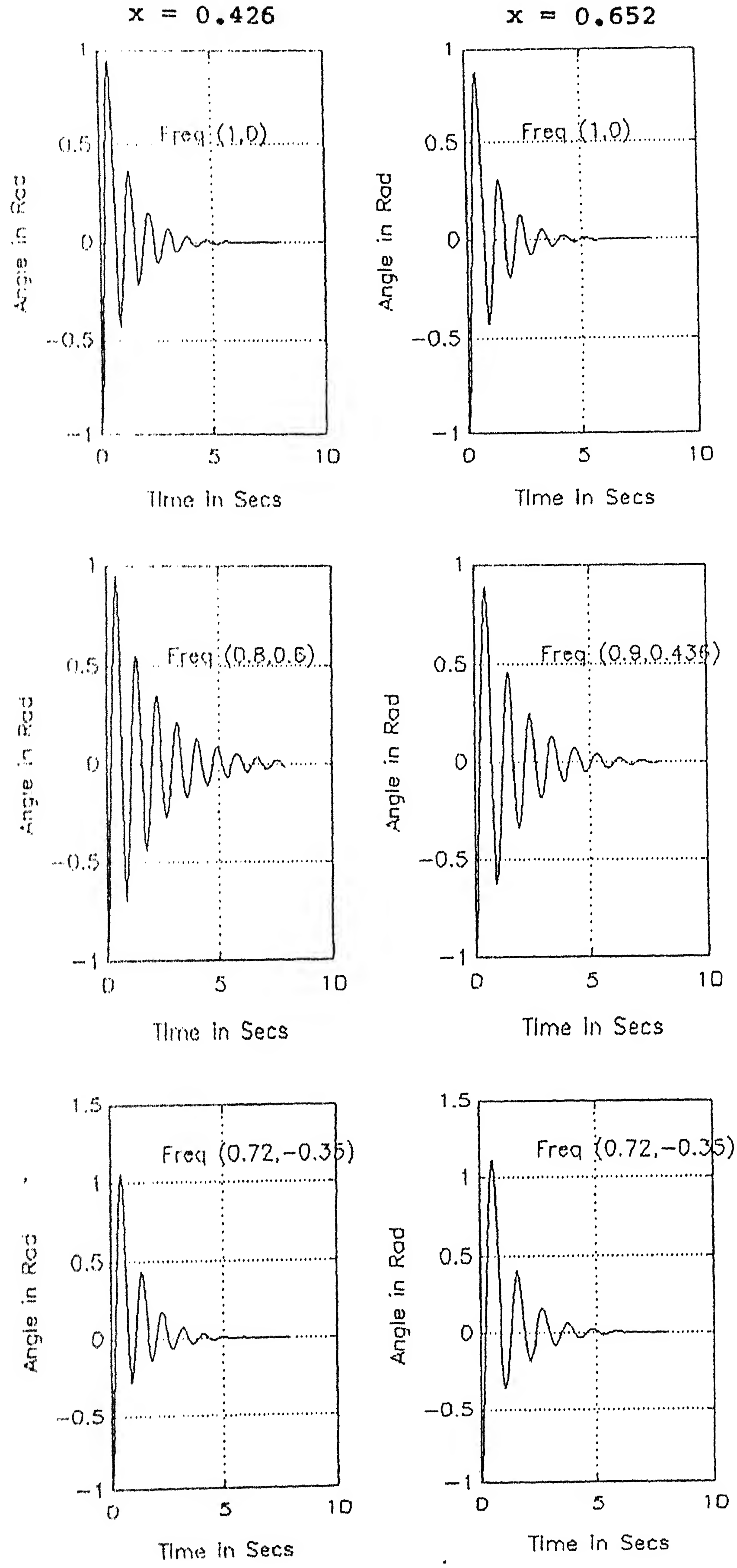


FIG. 4.4 INITIAL CONDITION RESPONSE WITH SVS BUS FREQUENCY SIGNAL.

Table 4.7 Numerical results with CIF signal for a weak system.

Input signal to SVS auxiliary controller : CIF

System strength : x = 0.652

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

-134.395228	
- .333149	+j10.614631
- .333149	-j10.614631
- 2.543204	
-102.318143	

Assigned poles

-1.5+/-j10.61

Auxiliary controller transfer function

$$F(s) = \frac{9.99999E{-}03\ s^2 + .77365403568\ s + 1.192203049E{-}02}{1.0\ s^2 + 7.95800E{-}02\ s + 1.52000E{-}03}$$

Closed loop eigenvalues

- 96.021998008	+j35.756830555
- 96.021998008	-j35.756830555
- 1.500000000	-j10.609983430
- 1.500000000	+j10.609983430
- 8.320863786	+j 6.136329435
- 8.320863786	-j 6.136329435
- 4.600594921	

Range of robustness on 1.0 p.u. MVA semicircle

(.85, 0.526783)
to
(.98, -0.198997)

Table 4.8 Numerical results with CIF signal for a strong system.

Input signal to SVS auxiliary controller : CIF

System strength : x = 0.426

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

- 96.967804	+j12.967576
- 96.967804	-j12.967576
- .378807	+j11.668060
.378807	-j11.668060
- 2.358224	

Assigned poles

-1.5+/-j11.668058

Auxiliary controller transfer function

$$F(s) = \frac{9.99999E{-}03\ s^2 + .7066347477\ s + 2.050569805048E{-}02}{1.0\ s^2 + 7.95800E{-}02\ s + 1.5200E{-}03}$$

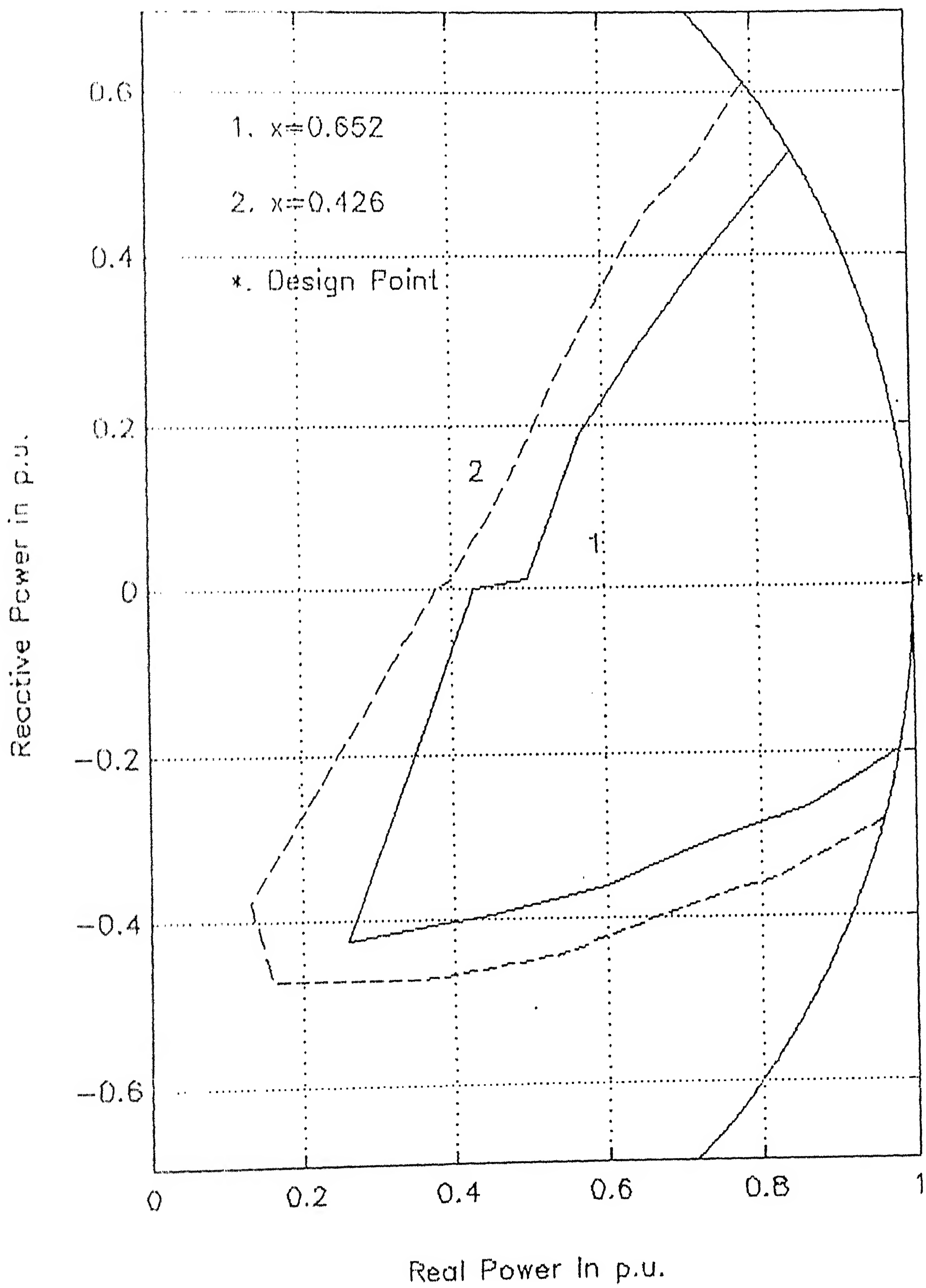
Closed loop eigenvalues

- 85.382266436	+j24.702898341
- 85.382266436	-j24.702898341
- 1.500000000	+j11.599977057
- 1.500000000	-j11.599977057
- 5.603801064	
- 8.000964012	+j 3.501394973
- 8.000964012	-j 3.501394973

Range of robustness on 1.0 p.u. MVA semicircle

(.79, 0.613107)
to
(.96, -0.280000)

Fig.4.5Regions of Robustness with CIF (Weak & Strong Systems)



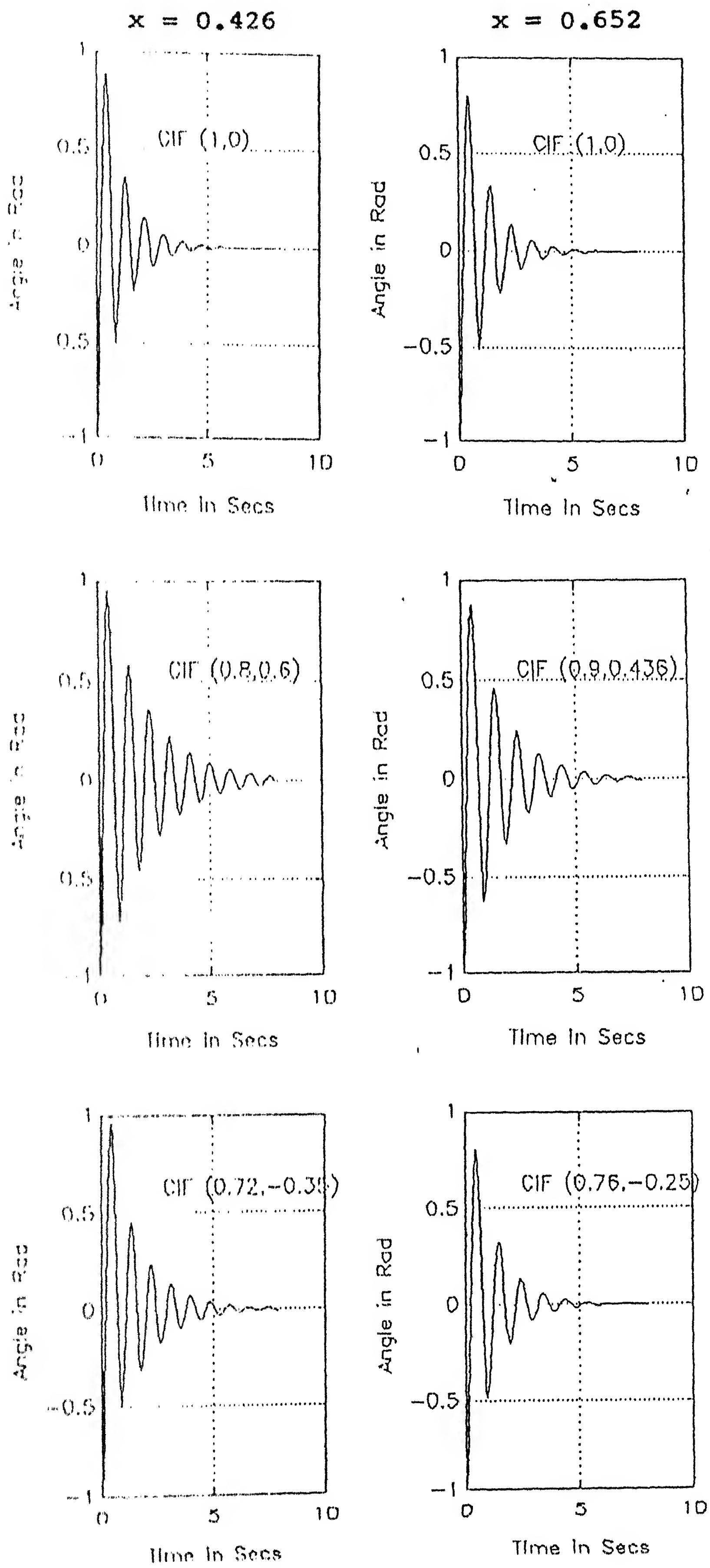


FIG. 4.6 INITIAL CONDITION RESPONSE WITH CIF SIGNAL.

Table 4.9 Numerical results with a combination of line power and SVS bus frequency signals for a weak system.

Input signal to SVS auxiliary controller : Line power and SVS bus frequency

System strength : x = 0.652

Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

-134.395228
- .333149 +j10.614631
- .333149 -j10.614631
- 2.543204
-102.318143

Assigned poles

-1.5+/-j10.61
-102.3
-2.54

Auxiliary controller transfer functions

$$F_1(s) = \frac{9.99999E-03\ s^2 - 3.719346695E-03\ s + 8.230537567E-05}{1.0\ s^2 + 7.95800E-02\ s + 1.52000E-03}$$

$$F_2(s) = \frac{-9.99999E-03\ s^2 - 4.4760677025E-02\ s + -1.17196882E-03}{1.0\ s^2 + 7.95800E-02\ s + 1.52000E-03}$$

Closed loop eigenvalues

-146.4200
-102.3000
- 8.4465 +j 8.6503
- 8.4465 -j 8.6503
- 1.5000 +j10.6100
- 1.5000 -j10.6100
- 2.5400

Range of robustness on 1.0 p.u. MVA semicircle

(.84, 0.542586)
to
(.95, -0.312250)

Table 4.10 Numerical results with a combination of line power and SVS bus frequency signal for a strong system.

Input signal to SVS auxiliary controller : line power and SVS bus frequency

System strength : x = 0.426
Design point : P = 1.0, Q = 0.0

Eigenvalues with only SVS voltage regulator

- 96.967804	+j12.967576
- 96.967804	-j12.967576
- .378807	+j11.668060
- .378807	-j11.668060
- 2.358224	

Assigned poles

-1.5+/-j11.66
-100
-2.36

Auxiliary controller transfer functions

$$F_1(s) = \frac{9.99999E-03\ s^2 - 4.3363497E-03\ s + 2.44459756E-04}{1.0\ s^2 + 7.95800E-02\ s + 1.5200E-03}$$

$$F_2(s) = \frac{-9.99999E-03\ s^2 - 5.7813503558\ s - 4.64747332265E-03}{1.0\ s^2 + 7.95800E-02\ s + 1.5200E-03}$$

Closed loop eigenvalues

-115.1600	
- 99.9710	
- 8.4239	+j 8.5781
- 8.4239	-j 8.5781
- 1.5000	+j11.6600
- 1.5000	-j11.6600
- 2.3600	

Range of robustness on 1.0 p.u. MVA semicircle

(.79, 0.613107)
to
(.91, -0.414608)

Tables 4.3 and 4.4 show the eigenvalues with only SVS voltage regulator, assigned eigenvalues, auxiliary controller transfer function and closed loop eigenvalues for weak and strong systems respectively for line power signal. Fig. 4.1 shows the regions of robustness for these two cases. The performance of the auxiliary controller is tested in the time domain by initial condition response at various test points in P-Q plane. Initial condition response for angular position of rotor is shown in Fig. 4.2.

Tables 4.5 and 4.6 show the eigenvalues with only SVS voltage regulator, assigned poles, auxiliary controller transfer function and closed loop eigenvalues for SVS bus frequency signal for weak and strong systems respectively. Fig. 4.3 shows the regions of robustness for these two cases. Initial condition response plots for angular position of rotor are shown in Fig. 4.4

Table 4.7 and 4.8 show the eigenvalues with only SVS voltage regulator, assigned poles, auxiliary controller transfer function and closed loop eigenvalues for CIF signal for weak and strong systems respectively. Fig. 4.5 shows the regions of robustness for these two cases. Initial condition response plots for angular position of rotor are shown in Fig. 4.6.

Table 4.9 and 4.10 show the eigenvalues with only SVS voltage regulator, assigned poles, auxiliary controller transfer function and closed loop eigenvalues for a combination of line power and SVS bus frequency signals for weak and strong systems

respectively. Fig. 4.7 shows the regions of robustness for these two cases.

The plots of robustness regions indicate that the region obtained for a strong system is largerthan that for a weak system for all the signals used. Initial condition response plots indicate a satisfactory performance of the controllers in time domain.

Figs. 4.8 and 4.9 show the regions of robustness for different signals for weak and strong systems respectively. It is evident from these two figures that region of robustness obtained with SVS bus frequency signal is larger than the regions with other signals. This is true for both weak and strong systems. Therefore SVS bus frequency signal is the most efficacious signal from robustness considerations for both weak and strong systems.

Fig.4.7Regions of Robustness with Power+Freq. (Weak & Strong Systems)

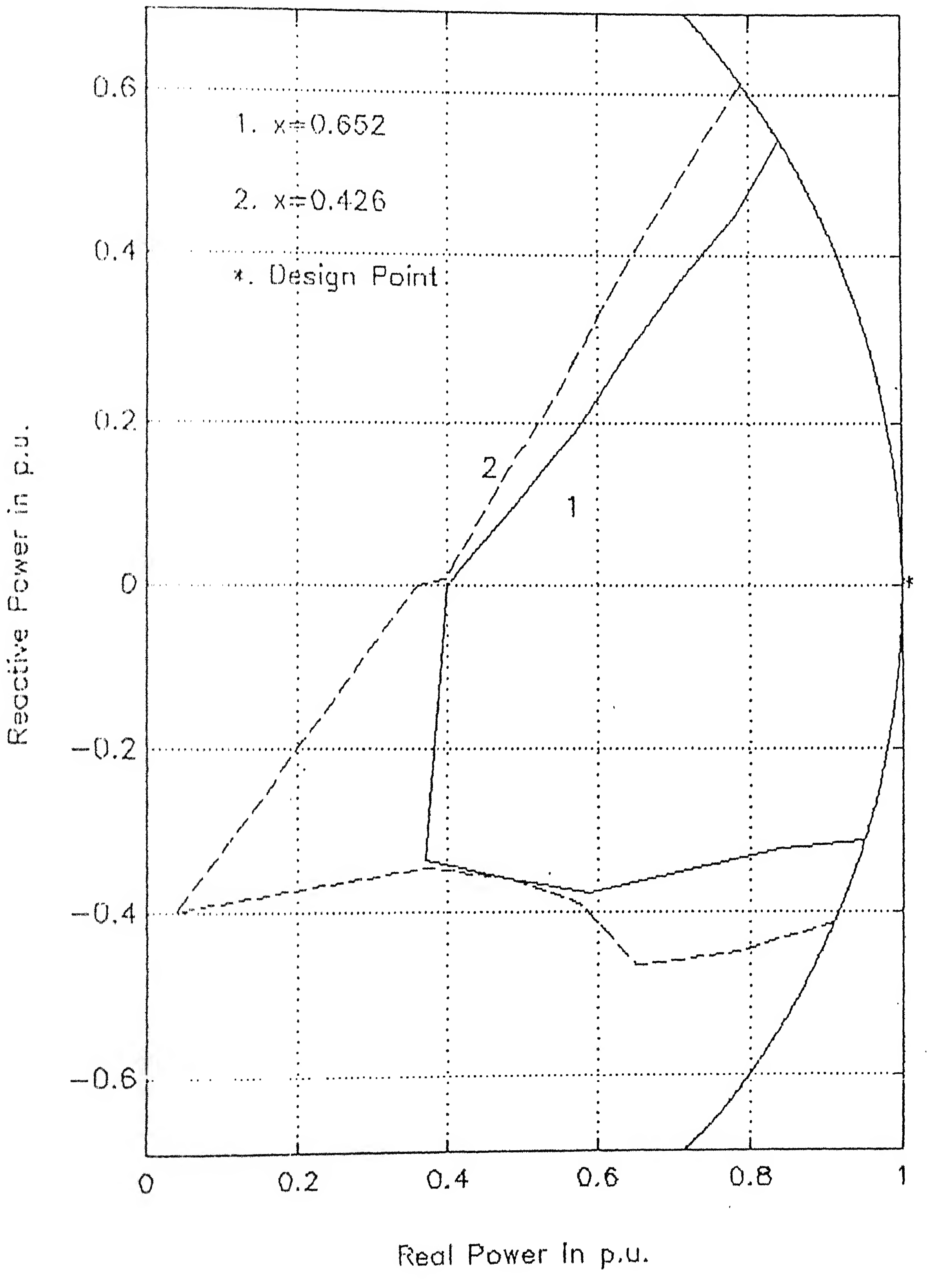


Fig.4.8Regions of Robustness for Weak System (x=0.652)

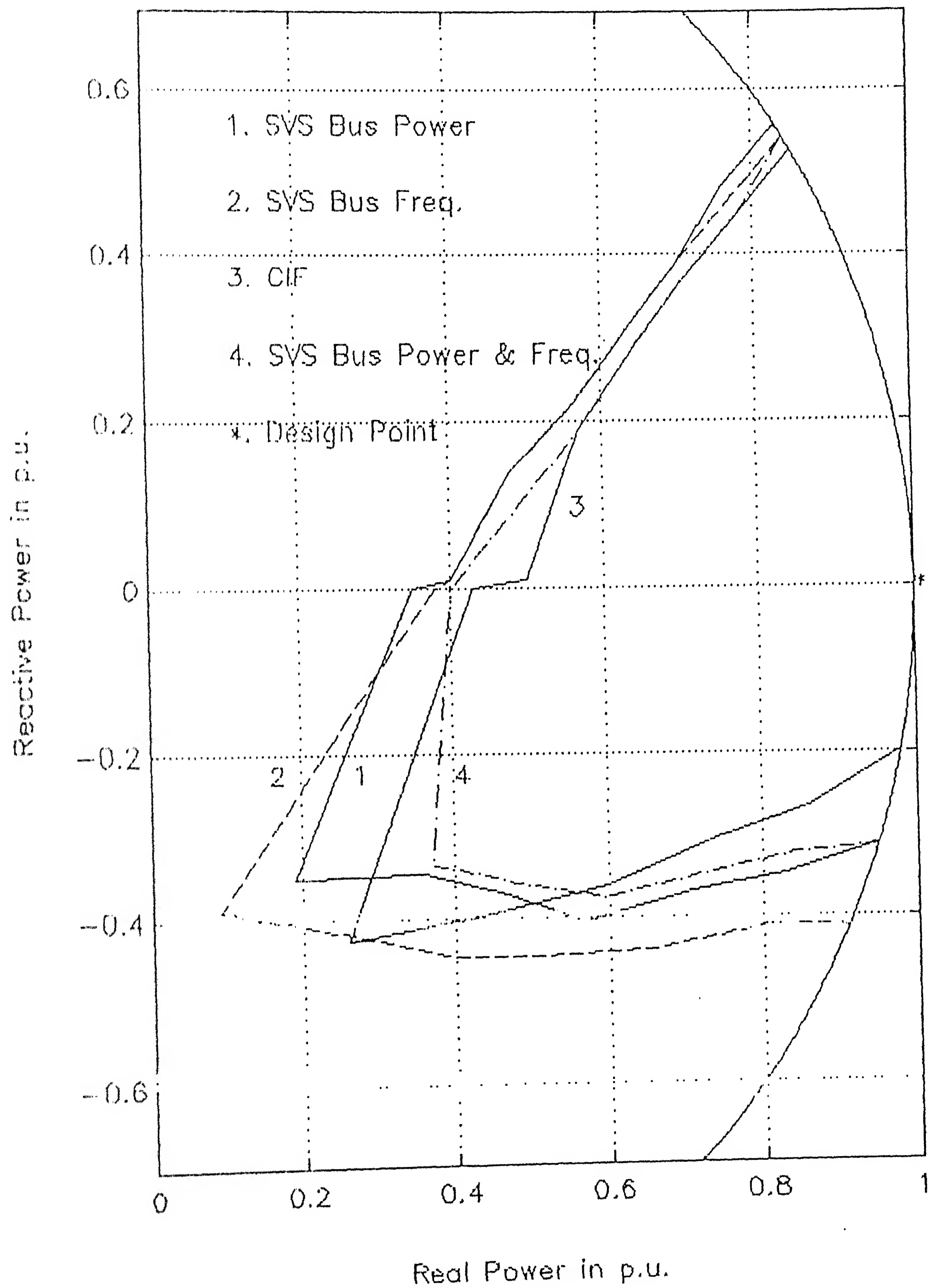
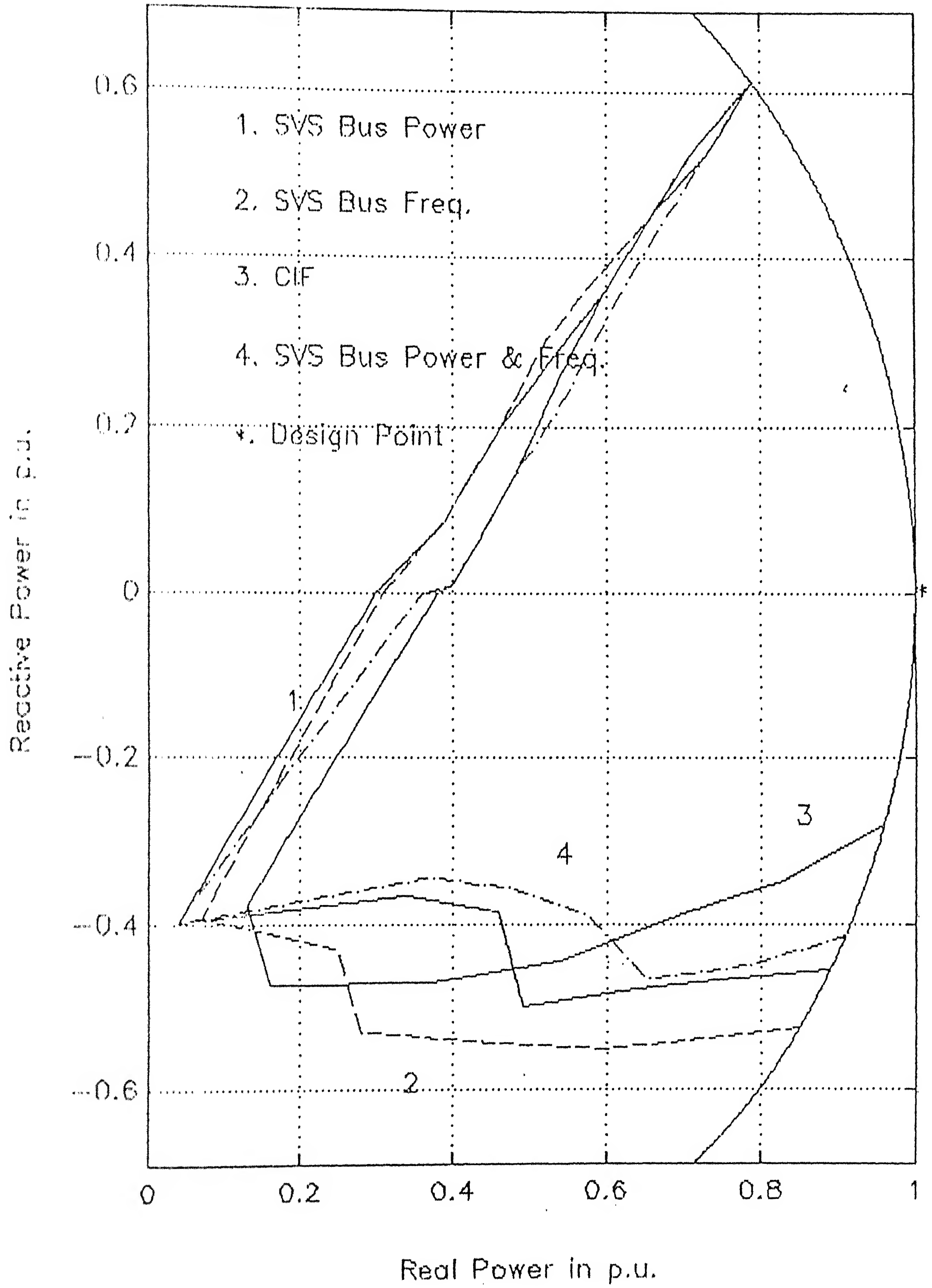


Fig.4.9Regions of Robustness for Strong System (x=0.426)



5. CONCLUSIONS

In this thesis the performance of SVS with an auxiliary controller in improving the dynamic stability of power system is studied and the most efficacious input signal to the controller is determined.

A new model for the power system is proposed. It is based on the Heffron-Phillips model and takes into account the SVS at the midpoint of the transmission line. The proposed model is valid for small perturbations around operating point.

An auxiliary controller is designed for SVS based on partial pole placement technique for dynamic output feedback. Line power, SVS bus frequency, Computed Internal Frequency and a combination of line power and SVS bus frequency are the input signals considered for auxiliary controller. A modified method based on the partial pole placement algorithm of Munro and Hirbod [33] is developed to design a controller for a system having a feedforward term in its state equation.

The effectiveness of the various signals is compared by plotting the regions of robustness with the signals for both weak and strong systems. It is clearly evident from the plots that the auxiliary controller with SVS bus frequency as the input signal gives the largest region of robustness in P-Q plane for both weak and strong systems. Thus SVS bus frequency is the most efficacious signal from robustness considerations in the improving the dynamic stability of power system.

SUGGESTIONS FOR FURTHER WORK

In this thesis the performance of SVS in improving dynamic stability of power system is studied by considering a single machine-infinite bus system. The actual power system is a multimachine system containing a number of machines and interconnections. The design of auxiliary controller for SVS and the study of performance of various signals in a multimachine system is an area for further work.

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APPENDIX - 1

POWER SYSTEM DATA AND LOAD FLOW RESULTS

The power system data considered are given below. The generator and network constants considered are same as given in [16] except for minor modifications. All values are in p.u.

GENERATOR PARAMETERS

$x_d = 1.7$ $\quad\quad$ $\tau'_{do} = 6.4$ s

$x_q = 1.64$ $\quad\quad$ $H = 4.2$ s

$x'_d = 0.345$ $\quad\quad$ $\omega_o = 314.14$

$x''_d = 0.235$

$x''_q = 0.235$

TRANSFORMER PARAMETERS

$x_t = 0.1$

TRANSMISSION LINE (500 kV, 100 kM, line Const. per Circuit)

$r_{line} = 0.0135$

$x_{line} = 0.452$

$B_{line} = 0.01147 \times 2$

AVR CONSTANTS

$K_A = 100.0$

$T_A = 0.01$ s

SVS VOLTAGE REGULATOR CONSTANT

$K_s = 20.0$

$T_s = 0.05$ s

LOAD FLOW RESULTS

Load flow results refer to the simplified model of the system shown in Fig. 2.2.

WEAK SYSTEM ($X^* = 0.652$)

$P = 1.0$, $Q = 0.0$

$V = 1.0$

$V_m = 1.0$ $\qquad \delta_m = 0.328463$

$V_t = 0.945873$ $\qquad \delta_t = 0.680357$

$Q_m = 0.507798$

STRONG SYSTEM ($x = 0.426$)

$P = 1.0$, $Q = 0.0$

$V = 1.0$

$V_m = 1.0$ $\qquad \delta_m = 0.213543$

$V_t = 0.979535$ $\qquad \delta_t = 0.432727$

$Q_m = 0.312764$

*x is the total inductive reactance of the transmission system.

APPENDIX - 2

EXPRESSIONS FOR GENERATOR INCREMENTAL CURRENT

Expressions for d-q components of generator current for small perturbations around operating point are derived here.

Referring to Fig. 2.2 to d-q components of the generator terminal voltage obtained by Park's transformation of network equations can be written as

$$v_{td} = -\sqrt{3}\ V \sin\delta + R\ (i_d - i_{1d}) + x(i_q - i_{1q}) + Ri_d + Xi_q \qquad (A2.1)$$

$$v_{tq} = -\sqrt{3}\ V \cos\delta + R\ (i_q - i_{1q}) - x(i_d - i_{1d}) + Ri_q - Xi_d \qquad (A2.2)$$

The expressions for SVS currents are

$$i_{1d} = (v_{tq} - Ri_q + Xi_d)\ B \qquad (A2.3)$$

$$i_{1q} = -(v_{td} - Ri_d - Xi_q)\ B \qquad (A2.4)$$

Simplifying eqns. (A2.1), (A2.2). (A2.3) and (A2.4) we get

$$v_{td}(1-BX) = -\sqrt{3}\ V \sin\delta + i_d(2R-2RBX) + i_q(BR^2-BX^2+2X) - v_{td}\ BR \qquad (A2.5)$$

$$v_{tq}(1-BX) = \sqrt{3}\ V \cos\delta + i_d(BX^2-BR^2-2X) + i_q(2R-2BRX) + v_{td}\ BR \qquad (A2.6)$$

From machine equations [22], we get,

$$v_{td} = -x_q i_q \tag{A2.7}$$

$$v_{tq} = e'_q + x'_d i_d \tag{A2.8}$$

Eliminating v_{td} and v_{tq} from eqns. (A2.5), (A2.6), (A2.7) and (A2.8), we get

$$[-x_q(1-BX) - (BR^2-BX^2+2X)]\ I_q - [(2R-2RBX) - (x'_d\ BR)]\ I_d$$

$$= -V \sin\delta - E'_q\ BR \tag{A2.9}$$

$$[-(2R-2RBX) + x_q\ BR]\ I_q + [x'_d\ (1-BX) - (BX^2-BR^2-2X)]\ I_d$$

$$= V \cos\delta - (1 - BX)\ E'_q \tag{A2.10}$$

Eqns. (A2.9) and (A2.10) are written for small perturbation around operating point as

$$[-x_q(1-BX) - (BR^2-BX^2+2X)]\ I_{q\Delta} + [-(2R-2RBX) + x'_d BR]\ I_{d\Delta}$$

$$= [-BR]\ E'_{q\Delta} + [-V \cos\delta]\ \delta_\Delta - [(x_q\ X\ I_q) - (R^2 - X^2)\ I_q$$

$$+ 2RXI_d + x'_d RI_d + E'_q R]\ B_\Delta \tag{A2.11}$$

$$[-(2R-2RBX) + x_q BR]\ I_{q\Delta} + [x'_d\ (1 - BX) - (BX^2-BR^2-2X)]\ I_{d\Delta}$$

$$= [-(1-BX)]\ E'_{q\Delta} + [-V \sin\delta]\ \delta_\Delta - [2R\ X\ I_q + x_q\ R\ I_q$$

$$+ x'_d X I_d + (X^2-R^2)\ I_d - XE'_q]\ B_\Delta \qquad (A2.12)$$

Eqns. (A2.11) and (A2.12) are written as

$$[C]\begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} = [D]\begin{bmatrix} E'_{q\Delta} \\ \delta_\Delta \\ B_\Delta \end{bmatrix} \qquad (A2.13)$$

where

$$[C] = \begin{bmatrix} -(2R-2RBX) + x'_d BR & -x_q(1-BX) - (BR^2-BX^2+2X) \\ x'_d(1-BX) - (BX^2-BR^2-2X) & -(2R-2BRX) + x_q BR \end{bmatrix}$$

$$[D] = \begin{bmatrix} -BR & -V\cos\delta & -[(x_q X I_q) - (R^2-X^2)I_q + 2RXI_d + x'_d RI_d + E'_q R] \\ -(1-BX) & -V\sin\delta & -[2RXI_q + x_q RI_q - x'_d XI_d - (X^2-R^2)\ I_d - XE'_q] \end{bmatrix}$$

Eqn. (A2.13) is written as

$$\begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} = [G]\begin{bmatrix} E'_{q\Delta} \\ \delta_\Delta \\ B_\Delta \end{bmatrix} . \qquad (A2.14)$$

where

$$[G] = [C]^{-1} [D]$$

Eqn. (A2.14) can be written as

$$\begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \end{bmatrix} \begin{bmatrix} E'_{q\Delta} \\ \delta_{\Delta} \\ B \end{bmatrix} \tag{A2.15}$$

APPENDIX - 3

EXPRESSIONS FOR K-COEFFICIENTS

Expressions for coefficients K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8 and K_9 are derived on the lines of Heffron-Philips model.

E'_q Equation

The d axis stator emf E_{FD} corresponding to the field voltage and the E'_q are related as [22],

$$E_{FD} = (1 + \tau'_{do}\, s)\, E'_q - (x_d - x'_d)\, I_d \qquad (A3.1)$$

Eqn. (A3.1) is written for small perturbations around operating point as

$$E_{FD\Delta} = (1 + \tau'_{do\Delta}\, s)\, E'_{q\Delta} - (x_d - x'_d)\, I_{d\Delta} \qquad (A3.2)$$

Substituting for $I_{d\Delta}$ from eqn. (A2.15), we get

$$E_{FD\Delta} = [1 - (x_d - x'_d)G_{11}]\, E'_{q\Delta} + \tau'_{do}\, s\, E'_{q\Delta} - (x_d - x'_d)G_{12}\, \delta_\Delta$$

$$- (x_d - x'_d)G_{13}\, B_\Delta \qquad (A3.3)$$

Eqn. (A3.3) is written as

$$E_{FD\Delta} = \left[\frac{1}{K_3} + \tau'_{do}\, s \right] E'_{qo} + K_4\, \delta_\Delta + K_8\, B_\Delta \qquad (A3.4)$$

where

$$\frac{1}{K_3} = 1 - (x_d - x_d^{`})\ G_{11}$$

$$K_4 = -(x_d - x_d^{`})\ G_{12}$$

$$K_5 = -(x_d - x_d^{`})\ G_{13}$$

Eqn. (A3.4) is written as

$$E_{q\Delta}^{`} = \left[\frac{K_3}{1+K_3\ \tau_{do}^{`}\ s}\right] E_{FD\Delta} - \left[\frac{K_3 K_4}{1+K_3\ \tau_{do}^{`}\ s}\right] \delta_\Delta - \left[\frac{K_3 K_8}{1+K_3\ \tau_{do}^{`}\ s}\right] B_\Delta$$

(A3.5)

<u>Torque Equation</u>

The expression for generator electrical torque is

$$T_e = 3\ [V_d\ I_d + V_q\ I_q] \tag{A3.6}$$

Substituting for v_d and v_q from eqn. (A2.7) and (A2.8) and writing the equation for small perturbations around operating point we get

$$T_{e\Delta} = I_{qo}\ E_{q\Delta}^{`} + E_{qao}\ I_{q\Delta} - (x_q - x_d^{`})\ I_{d\Delta} \tag{A3.7}$$

where

$$E_{qao} = E_{qo}^{`} - (x_q - x_d^{`})\ I_{do}$$

Substituting for $I_{q\Delta}$ and $I_{d\Delta}$ from eqn. (A2.15) we get,

$$T_{e\Delta} = 3[E_{qao}\ G_{22} - (x_q - x_d^{`})\ I_{qo}\ G_{12}]\ \delta_\Delta$$

$$+ 3[I_{qo} + E_{qao}\ G_{21} - (x_q - x_d^{`})\ I_{qo}\ G_{11}]\ E_{q\Delta}^{`}$$

$$+ 3[E_{qao}\ G_{23} - (x_q - x_d^{`})\ I_{qo}\ G_{13}]\ B_\Delta \tag{A3.8}$$

Eqn. (A3.8) is written as

$$T_{e\Delta} = K_1 \delta_\Delta + K_2 E'_{q\Delta} + K_7 B_\Delta \tag{A3.9}$$

where

$$K_1 = 3[E_{qao} G_{22} - (x_q - x'_d) I_{qo} G_{12}]$$

$$K_2 = 3[I_{qo} + E_{qao} G_{21} - (x_q - x'_d) I_{qo} G_{11}]$$

$$K_3 = 3[E_{qao} G_{23} - (x_q - x'_d) I_{qo} G_{13}]$$

Terminal Voltage Equation

The generator terminal voltage in terms of its d-q components is given as

$$V_t^2 = V_d^2 + V_q^2 \tag{A3.10}$$

Eqn. (A3.10) is written for small perturbations around operating point as

$$V_{t\Delta} = \left(\frac{V_{do}}{V_{to}} \right) V_{d\Delta} + \left(\frac{V_{qo}}{V_{to}} \right) V_{q\Delta} \tag{A3.11}$$

From eqns. (A2.7), (A2.8) and (A3.11) we get

$$V_{t\Delta} = -\left(\frac{V_{do}}{V_{to}} \right) x_q I_{q\Delta} + \left(\frac{V_{qo}}{V_{to}} \right) (x'_d I_{d\Delta} + E'_{q\Delta}) \tag{A3.12}$$

Substituting for $I_{d\Delta}$ and I_{qo} from eqn. (A2.15) we get

$$V_{t\Delta} = K_5 \delta_\Delta + K_6 E'_{q\Delta} + K_9 B_\Delta \tag{A3.13}$$

where

$$K_5 = \left[-\left(\frac{V_{do}}{V_{to}}\right) x_q \; G_{22} + \left(\frac{V_{qo}}{V_{to}}\right) x'_d \; G_{12}\right]$$

$$K_6 = \left[\frac{V_{qo}}{V_{to}} - \left(\frac{V_{do}}{V_{to}}\right) x_q \; G_{21} + \left(\frac{V_{qo}}{V_{to}}\right) x'_d \; G_{11}\right]$$

$$K_9 = \left[-\left(\frac{V_{do}}{V_{to}}\right) x_q \; G_{23} + \left(\frac{V_{qo}}{V_{to}}\right) x'_d \; G_{13}\right]$$

APPENDIX - 4

DERIVATIONS OF THE EXPRESSIONS FOR SIGNALS USED

Expression for Midpoint Voltage Signal

The d-q components of the mid point voltage of treansmission line are

$$v_{md} = v_{td} - Ri_d - Xi_q \tag{A4.1}$$

$$v_{mq} = v_{tq} - Ri_q + Xi_d \tag{A4.2}$$

Eqns. (A4.1) and (A4.2) are written for small perturbations around operating point as

$$v_{md\Delta} = v_{td\Delta} - Ri_{d\Delta} - Xi_{q\Delta} \tag{A4.3}$$

$$v_{mq\Delta} = v_{tq\Delta} - Ri_{q\Delta} - Xi_{d\Delta} \tag{A4.4}$$

From eqns. (A2.7) and (A2.8),

$$v_{td} = - x_q \; i_q \tag{A4.5}$$

$$v_{tq} = e'_q + x'_d \; i_d \tag{A4.6}$$

Writing eqns. (A4.5) and (A4.6) for small perturbations around operating condition we get

$$v_{td\Delta} = - x_q \; i_{q\Delta} \tag{A4.7}$$

$$v_{tq\Delta} = e'_{q\Delta} + x'_d \; i_{d\Delta} \tag{A4.8}$$

Simplifying eqns. (A4.3), (A4.4), (A4.7) and (A4.8) we get

$$v_{md\Delta} = (-x_q - X)\, i_{q\Delta} - R i_{d\Delta} \qquad (A4.9)$$

$$v_{mq\Delta} = -R i_{q\Delta} + (X + x_d')\, i_{d\Delta} + e_q' \qquad (A4.10)$$

Substituting (A4.9) and (A4.10) are written as

$$\begin{bmatrix} V_{md\Delta} \\ V_{mq\Delta} \end{bmatrix} = \begin{bmatrix} -R & -x_q-X \\ X+x_d' & -R \end{bmatrix} \begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E_{q\Delta}' \qquad (A4.11)$$

Substituting for $I_{d\Delta}$ and $I_{q\Delta}$ from eqn. (A2.15) we get

$$\begin{bmatrix} V_{md\Delta} \\ V_{mq\Delta} \end{bmatrix} = \left\{ [F]\,[G] + [H] \right\} \begin{bmatrix} E_{q\Delta}' \\ \delta_{\Delta} \\ B_{\Delta} \end{bmatrix} \qquad (A4.12)$$

where

$$[F] = \begin{bmatrix} -R & -x_q-X \\ X+x_d' & -R \end{bmatrix}$$

$$[H] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The mid point voltage equation for small perturbations around operating point is

$$\begin{bmatrix} \dfrac{V_{mdo}}{|V_{mo}|} & \dfrac{V_{mqo}}{|V_{mo}|} \end{bmatrix} \begin{bmatrix} V_{md\Delta} \\ V_{mq\Delta} \end{bmatrix} \tag{A4.13}$$

Simplifying eqns. (A4.12) and (A4.13) we get

$$V_{m\Delta} = [T] \left\{ [F]\ [G] + [H] \right\} \begin{bmatrix} E'_{q\Delta} \\ \delta_{\Delta} \\ B_{\Delta} \end{bmatrix} \tag{A4.14}$$

where

$$[T] = \begin{bmatrix} \dfrac{V_{mdo}}{|V_{mo}|} & \dfrac{V_{mqo}}{|V_{mo}|} \end{bmatrix}$$

Eqn. (A4.14) is written as

$$V_{m\Delta} = \begin{bmatrix} R_1 & R_2 & R_3 \end{bmatrix} \begin{bmatrix} E'_{q\Delta} \\ \delta_{\Delta} \\ B_{\Delta} \end{bmatrix} \tag{A4.15}$$

<u>Expression for Line Power</u>

Line power is written as

$$P_m = v_{md}\, i_d + v_{mq}\, i_q \tag{A4.16}$$

Eqn. (A4.16) is written for small perturbations around operating point as

$$P_{m\Delta} = v_{md}\, i_{d\Delta} + i_d\, v_{md\Delta} + v_{mq}\, i_{q\Delta} + i_q\, v_{mq\Delta} \tag{A4.17}$$

Substituting for $i_{d\Delta}$ and $i_{q\Delta}$ from eqn. (A2.15) and for $v_{md\Delta}$ and $v_{mq\Delta}$ from eqn. (A4.12) we get,

$$P_{m\Delta} \left\{ [G_1]\ [G] + [G_2] \left\{ [F]\ [G] + [H] \right\} \right\} \begin{bmatrix} E'_{q\Delta} \\ \delta_{\Delta} \\ B_{\Delta} \end{bmatrix} \qquad \text{(A4.18)}$$

where

$$[G_1] = [\ 3V_{md} \quad 3V_{mq}\ ]$$

$$[G_2] = [\ 3I_d \quad 3\ I_q\ ]$$

Eqn. (A4.18) is written as,

$$P_{m\Delta} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix} \begin{bmatrix} E'_{q\Delta} \\ \delta_{\Delta} \\ B_{\Delta} \end{bmatrix} \qquad \text{(A4.19)}$$

<u>Expression for SVS Bus Frequency</u>

SVS bus voltage angle is

$$\delta_m = \tan^{-1} \left(\frac{V_{md}}{V_{mq}} \right) \qquad \text{(A4.20)}$$

SVS bus frequency is

$$\omega_m = \frac{d}{dt}\ \delta_m \qquad \text{(A4.21)}$$

From eqns. (A4.20) and (A4.21) we get

$$\omega_m = \frac{d}{dt} \tan^{-1} \left(\frac{V_{md}}{V_{mq}} \right) \qquad \text{(A4.22)}$$

Eqn. (A4.22) is written for small perturbations around operating point as

$$\omega_{m\Delta} = \begin{bmatrix} \dfrac{V_{mqo}}{|V_{mo}|^2} & \dfrac{-V_{mdo}}{|V_{mo}|^2} \end{bmatrix} \begin{bmatrix} \dfrac{d}{dt} V_{md\Delta} \\ \dfrac{d}{dt} V_{mq\Delta} \end{bmatrix} \qquad (A4.23)$$

Let

$$[T_1] = \begin{bmatrix} \dfrac{V_{mqo}}{|V_{mo}|^2} & \dfrac{-V_{mdo}}{|V_{mo}|^2} \end{bmatrix}$$

Simplifying eqns. (A4.12) and (A4.23) we get

$$\omega_{m\Delta} = \left\{ [T_1]\ [F]\ [G] + [H] \right\} \begin{bmatrix} \dot{E}'_{q\Delta} \\ \dot{\delta}_{\Delta} \\ \dot{B}_{\Delta} \end{bmatrix} \qquad (A4.24)$$

Eqn. (A4.24) is written as

$$\omega_{m\Delta} = [T_1] \left\{ [F]\ [G] + [H] \right\} [T_2] \begin{bmatrix} \dot{E}'_{q\Delta} \\ \dot{\omega}_{\Delta} \\ \dot{\delta}_{\Delta} \\ \dot{F}_{FD\Delta} \\ \dot{B}_{\Delta} \end{bmatrix} \qquad (A4.25)$$

where

$$[T_2] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Eqn. (A4.25) is written as

$$\omega_{m\Delta} = [T_3]\ \dot{\underline{x}} \tag{A4.26}$$

where $[T_3] = [T_1] \left\{ [F]\ [G] + [H] \right\} [T_2]$

Eqn. (A4.26) is written as,

$$\omega_{m\Delta} = [T_3] \left\{ A\ \underline{x} + \underline{b}_3\ u_c \right\} \tag{A4.27}$$

or

$$y_{c2} = [C_{c2}]\ \underline{x} + d_{c2}\ u_c$$

where
$$y_{c2} = \omega_{m\Delta}\quad \underline{x} = [E'_{q\Delta}\ \omega_\Delta\ \delta_\Delta\ E_{FD\Delta}\ B_\Delta]^t$$

$$C_{c2} = [T_3]\ [A]$$

$$d_{c2} = [T_3]\ \underline{b}_3$$

[A] and $\underline{b}_3$ are state and output matrices defined in Chapter 3.

<u>Expression for Computed Internal Frequency</u>

An expression for CIF is derived using the generator voltage behind subtransient reactance, line current and SVS bus voltage.

The generator terminal voltage in terms of voltage behind subtransient reactance is, [22]

$$V_{td} = - I_q\ x'' + E''_d \tag{A4.28}$$

$$V_{tq} = I_d\ x'' + E''_q \tag{A4.29}$$

The generator terminal voltage in terms of SVS bus voltage and line current is,

$$V_{td} = V_{md} + RI_d + XI_q \tag{A4.30}$$

$$V_{tq} = V_{mq} + RI_q - XI_d \tag{A4.31}$$

From eqns. (A4.28), (A4.29), (A4.30) and (A4.31) we get

$$E''_d = V_{md} + RI_d + (X + x'')\ I_q \tag{A4.32}$$

$$E''_q = V_{mq} + RI_q - (X + x'')\ I_d \tag{A4.33}$$

The internal voltage angle is

$$\delta_c = \tan^{-1}\left[\frac{E''_d}{E''_q}\right] \tag{A4.34}$$

Computed internal frequency is

$$\omega_c = \frac{d}{dt}\ \delta_c \tag{A4.35}$$

From eqn. (A4.34) and (A4.35) we get

$$\omega_c = \frac{d}{dt}\ \tan^{-1}\left[\frac{E''_d}{E''_q}\right] \tag{A4.36}$$

Eqn. (A4.36) is written for small perturbations around operating point as

$$\omega_{c\Delta} = \begin{bmatrix} \dfrac{E''_{qo}}{|E''_o|^2} & \dfrac{-E''_{do}}{|E''_o|^2} \end{bmatrix} \begin{bmatrix} \dfrac{d}{dt} & E''_{d\Delta} \\ \dfrac{d}{dt} & E''_{q\Delta} \end{bmatrix} \tag{A4.37}$$

Let

$$[S_1] = \begin{bmatrix} \dfrac{E''_{qo}}{|E''_o|^2} & \dfrac{-E''_{do}}{|E''_o|^2} \end{bmatrix}$$

Eqns. (A4.32) and (A4.33) are written for small perturbations around operating point as

$$\begin{bmatrix} E''_{d\Delta} \\ E''_{q\Delta} \end{bmatrix} = [S_2] \begin{bmatrix} V_{md\Delta} \\ V_{mq\Delta} \end{bmatrix} + [S_3] \begin{bmatrix} I_{d\Delta} \\ I_{q\Delta} \end{bmatrix} \qquad \text{(A4.38)}$$

where

$$[S_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[S_3] = \begin{bmatrix} R & X+x'' \\ -X-x'' & R \end{bmatrix}$$

Simplifying eqns. (A4.38), (A2.15) and (A4.12) we get

$$\begin{bmatrix} E''_{d\Delta} \\ E''_{q\Delta} \end{bmatrix} = [S_4] \begin{bmatrix} E'_{q\Delta} \\ \delta_\Delta \\ B_\Delta \end{bmatrix} \qquad \text{(A4.39)}$$

where

$$[S_4] = \Big\{ [S_2] \Big\{ [F]\ [G] + [H] \Big\} + [S_3]\ [G] \Big\}$$

Simplifying eqns. (A4.37) and (A4.39) we get

$$\omega_{c\Delta} = [S_1] \; [S_4] \; [S_5] \begin{bmatrix} \dot{E}'_{q\Delta} \\ \dot{\omega}_{\Delta} \\ \dot{\delta}_{\Delta} \\ \dot{E}_{FD\Delta} \\ \dot{B}_{\Delta} \end{bmatrix} \tag{A4.40}$$

where

$$[S_5] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Eqn. (A4.40) is written as

$$\omega_{c\Delta} = [S_6] \; \dot{\underline{x}} \tag{A4.41}$$

where

$$[S_6] = [S_1] \; [S_4] \; [S_5]$$

or

$$\omega_{c\Delta} = [S_6] \; [A \; \underline{x} + \underline{b}_3 \; u_c] \tag{A4.42}$$

Eqn. (A4.42)is written as

$$y_{c3} = C_{c3} \; \underline{x} + d_{c3} \; u_c \tag{A4.43}$$

where

$$\underline{x} = [E'_{q\Delta} \;\; \omega_{\Delta} \;\; \delta_{\Delta} \;\; E_{FD\Delta} \;\; B_{\Delta}]^t$$

$$C_{c3} = [S_6] \; [A]$$

$$d_{c3} = [S_6] \; \underline{b}_3$$

APPENDIX - 5

SYSTEM EQUATIONS WITH SVS VOLTAGE REGULATOR

The state and output equations for the machine and excitation system are given in Chapter 2. These equations are

$$\dot{x}_1 = A_1 \underline{x}_1 + \underline{b}_1 u_1 \qquad (A5.1)$$

$$y_1 = C_1 \underline{x}_1 + d_1 u_1 \qquad (A5.2)$$

where

$$\underline{x}_1 = [E'_{q\Delta} \ \omega_\Delta \ \delta_\Delta \ E_{FD\Delta}]$$

$$u_1 = B_\Delta, \qquad y_1 = V_{m\Delta}$$

The state and output equations for SVS voltage regulator as given in Chapter 2 are,

$$\dot{x}_2 = A_2 x_2 + b_2 u_2 \qquad (A5.3)$$

$$y_2 = C_2 x_2 \qquad (A5.4)$$

where

$$x_2 = y_2 = B_\Delta$$

$$u_2 = - V_{m\Delta}$$

Let $\underline{u} = [u_1 \quad u_2]^t$

$\underline{y} = [y_1 \quad y_2]^t$

then

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{A5.5}$$

Let $F_c = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

then $$\underline{u} = F_c \; y \tag{A5.6}$$

Let $\underline{x} = [\underline{x}_1 \quad x_2]^t$

$$A_c = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad C_c = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$B_c = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \qquad D_c = \begin{bmatrix} d_1 & 0 \\ 0 & 0 \end{bmatrix}$$

Eqns. (A5.1), (A5.2), (A5.3) and (A5.4) are written as

$$\dot{\underline{x}} = A_c \underline{x} + B_c \underline{u} \tag{A5.7}$$

$$\underline{y} = C_c \underline{x} + D_c \underline{u} \tag{A5.8}$$

From eqns. (A5.6) and (A5.8) we get

$$\underline{u} = F_c \, [C_c \underline{x} + D_c \underline{u}] \tag{A5.9}$$

Simplifying eqn. (A5.9) we get

$$\underline{u} = [I - F_c D_c]^{-1} F_c C_c \underline{x} \qquad (A5.10)$$

From eqns. (A5.10) and (A5.7)

$$\dot{\underline{x}} = \left\{ A_c + B_c \left[I - F_c D_c\right]^{-1} F_c C_c \right\} \underline{x} \qquad (A5.11)$$

Let $A = A_c + B_c [I - F_c D_c]^{-1} F_c C_c$

then $\dot{\underline{x}} = A \underline{x}$ (A5.12)

APPENDIX - 6

POLE ASSIGNMENT WITH OUTPUT FEEDBACK

The technique given here is based on the work of Munro and Hirbod [33]. The method presented in [33] provides a technique for the design of full-rank compensators for multivariable linear systems. Here the technique is restricted to single input linear systems

$$p\,\underline{x} = [A]\,\underline{x} + \underline{b}\,u \tag{A6.1}$$

and

$$\underline{y} = [C]\,\underline{x} \tag{A6.2}$$

where, $x \in R^n$, $u \in R^1$ and $\underline{y} \in R^r$ are respectively vectors of state, input and output variables. The r x 1 open-loop transfer function matrix G(s) is

$$G(s) = C(sI - A)^{-1}\,b = \frac{1}{\Delta_o(s)}\begin{bmatrix} N_1(s) \\ \cdot \\ \cdot \\ \cdot \\ N_r(s) \end{bmatrix} \tag{A6.3}$$

where

$$\Delta_o(s) = s^n + \alpha_1 s^{n-1} + \ldots + \alpha_n \tag{A6.4}$$

and

$$N_i(s) \quad \beta_{i1}s^{n-1} + \beta_{i2}s^{n-2} + \ldots + \beta_{in} \tag{A6.5}$$

$\Delta_o(s)$ is the characteristic polynomial of G(s). Consider the output feedback law

$$u(s) = u_r(s) - F(s) \cdot y(s) \tag{A6.6}$$

where $u_r \in R^1$ represents the reference input.

The problem can be defined as the determination of 1 x r dynamic feedback compensator, F(s), such that the closed-loop system

$$H(s) = [I + G(s) F(s)]^{-1} G(s) \tag{A6.7}$$

has a desired set of eigenvalues. The kth order compensator F(s) has the form

$$F(s) = \frac{1}{\Delta_c(s)} [M_1(s), M_2(s), \ldots, M_r(s)] \tag{A6.8}$$

where

$$\Delta_c = s^k + \gamma_1 s^{k-1} + \gamma_2 s^{k-2} + \ldots. + \gamma_k \tag{A6.9}$$

and

$$M_i(s) = \theta_{io} s^k + \theta_{i1} s^{k-1} \ldots + \theta_{ik} \tag{A6.10}$$

$\Delta_c(s)$ is the characteristic polynomial of compensator transfer function F(s).

The resulting closed-loop system characteristic polynomial $\Delta_d(s)$, defined as :

$$\Delta_d(s) \quad s^{n+k} + d_1 s^{n+k-1} + \ldots + d_{n+k} \tag{A6.11}$$

can be written as [33]

$$\Delta_d(s) = \Delta_o(s).\Delta_c(s) + \sum_{i=1}^{r} N_i(s).M_i(s) \tag{A6.12}$$

COMPLETE POLE ASSIGNMENT

The problem of pole assignment can be defined as : given $\Delta_o(s)$, $N_1(s)$, ..., $N_r(s)$ and a desired set of closed-loop poles, find $\Delta_c(s)$, $M_1(s)$,..., $M_r(s)$ of lowest degree which satisfy eqn. (A6.12). Equating coefficients of like powers of s in eqn. (A6.12), we get

$$[X_k]\ \underline{p}_k = \underline{\delta}_k \tag{A6.13}$$

where

$$[X_k] = \left[\begin{array}{cccc:ccc:ccc}
1 & 0 & \cdots & 0 & \beta_{11} & \cdots & 0 & \beta_{r1} & \cdots & 0 \\
\alpha_1 & 1 & \cdots & . & \beta_{12} & \cdots & 0 & \beta_{r2} & & 0 \\
\alpha_2 & \alpha_1 & & & . & \cdots & & & & \\
. & \alpha_2 & & 1 & & & \beta_{11} & & & \beta_{r1} \\
. & . & & & & & & & & \\
. & . & & \alpha_1 & \beta_{1n} & & \beta_{12} & \beta_{rn} & & \beta_{r2} \\
\alpha_n & . & & \alpha_2 & 0 & & & 0 & & \\
0 & \alpha_n & & & 0 & & & 0 & & \\
0 & 0 & & & & & & & & \\
. & & & & & & & & & . \\
. & & & & & & & & & . \\
. & & & & & & & & & . \\
0 & 0 & & \alpha_n & 0 & & \beta_{1n} & 0 & & \beta_{rn}
\end{array}\right]$$

K columns K+1 columns K+1 columns

$$\underline{p}_k = [\gamma_1\ \gamma_2\ \cdots\ \gamma_k \ \vdots\ \vdots\ \theta_{10}\ \cdots\ \theta_{1k}\ \vdots\ \cdots\ \vdots\theta_{ro}\ \cdots\ \theta_{rk}]^t$$

and

$$\underline{\delta}_k = [d_1-\alpha_1)\ \vdots\ (d_2-\alpha_2)\ \vdots\cdots\ \vdots\ (d_n-\alpha_n)\ \vdots\ d_{n+1}\ \cdots\ d_{n+k}]^t$$

Equation (A6.13) is a set of (n+k) equations in [(k+1)r+k] unknown parameters of F(s). The difference vector δ_k contains the coefficients of the polynomial

$$\Delta_d^{`}(s) = \Delta_d(s) - \Delta_o(s)\ .\ s^k \qquad (A6.14)$$

A necessary and sufficient codnition for the existence of solution of eqn. (A6.13) is

$$\text{Rank } [X_k] = \text{Rank } [X_k,\ \delta_k] \qquad (A6.15)$$

and a solution of eqn. (A6.13) is

$$\underline{p}_k = [X_k^{g1}]\ \underline{\delta}_k \qquad (A6.16)$$

where X_k^{g1} is the generalized inverse of X_k such that

$$[X_k]\ [X_k^{g1}]\ [X_k] = [X_k]$$

PARTIAL POLE ASSIGNMENT

Consider the case when q poles are to be assigned, t = (n + k - q) poles are allowed to assume arbitrary locations. For this case the closed-loop system characteristic polynomial $\Delta_d(s)$ has the form

$$\Delta_d(s) = (s^q + d_1 s^{q-1} + \ldots d_q)(s^t + e_1 s^{t-1} + \ldots + e_t) \quad \text{(A6.17)}$$

where $d_1, d_2, \ldots, d_q$ are specified and $e_1, e_2, \ldots, e_t$ are to be determined. Here the difference vector δ_k an be obtained by equating the coefficients of like powers of` s in (A6.17) and (A6.12) as

$$\underline{\delta}_k = \underline{\delta}_k^` + e_1 \underline{\delta}_1 + \ldots + e_t \underline{\delta}_t \quad \text{(A6.18)}$$

where the vectors $\delta_k^`$ and δ_i, $(i = 1, \ldots, t)$ contain respectively the coefficients of the polynomials

$$\Delta_d''(s) = \Delta_q(s).s^t - \Delta_o(s).s^k \quad \text{(A6.19)}$$

and

$$\Delta_i''(s) = \Delta_q(s).s^{t-i}, \quad (i = 1, \ldots, t) \quad \text{(A6.20)}$$

Eqn. (A6.13) in this case will take the form

$$[X_k]\, \underline{p}_k = \underline{\delta}_k^` + [D]\, \underline{e} \quad \text{(A6.21)}$$

where

$$[D] = [\underline{\delta}_1, \underline{\delta}_2, \ldots.\ \underline{\delta}_t]$$

and

$$\underline{e} = [e_1, e_2, \ldots, e_t]^t$$

Eqn. (A6.21) can be rearranged as

$$[X_k^`]\, \underline{p}_k^` = \underline{\delta}_k^` \quad \text{(A6.22)}$$

where

$$[X_k^`] = [X_k, -D]$$

and (A6.23)

$$\underline{p}_k^` = [p_k^t, e^t]^t$$

Eqn. (A6.22) is a set of (n+k) simultaneous algebraic equations in [r(k+1) + k + t] unknowns. Vector $p_k^`$ contains parameters of F(s) and the coefficients of polynomial of unassigned closed-loop poles. The necessary and sufficient condition for the existence of solution of eqn. (A6.22) is

$$\text{Rank } [X_k^`] = \text{Rank } [X_k^`, \delta_k^`].$$

APPENDIX - 7

STATE EQUATIONS FOR SVS AUXILIARY CONTROLLER

A block diagram for SVS auxiliary controller is shown in Fig. A7.1.

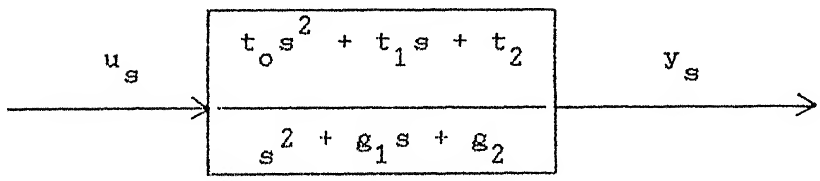

Fig. A7.1

Let

$$F(s) = \frac{t_o s^2 + t_1 s + t_2}{s^2 + g_1 s + g_2} \tag{A7.1}$$

Let $\quad \dfrac{t_1}{t_o} = \alpha_1, \quad \dfrac{t_2}{t_o} = \alpha_2$

Eqn. (A7.1) is written as

$$F(s) = \frac{t_o[s^2 + \alpha_1 s + \alpha_2]}{s^2 + g_1 s + g_2} \tag{A7.2}$$

$$F(s) = t_o \left[1 + \frac{(\alpha_1 - g_1)s + (\alpha_2 - g_2)}{s^2 + g_1 s + g_2} \right] \tag{A7.3}$$

Eqn. (A7.3) is represented by the block diagram shown in Fig.A7.2

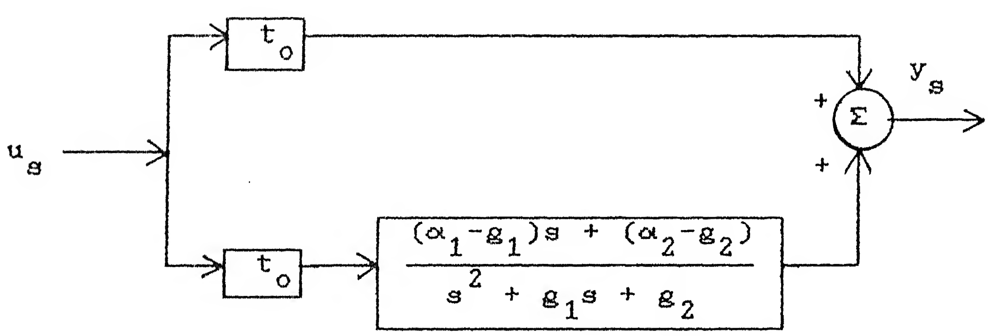

Fig. A7.2.

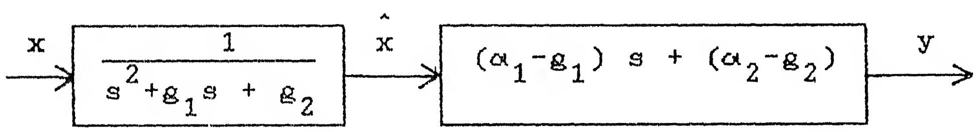

Fig. A7.3.

Referring to Fig. A7.3,

$$\ddot{\hat{x}} + g_1 \dot{\hat{x}} + g_2 \hat{x} = x \qquad \text{(A7.4)}$$

Let $\eta_1 = \hat{x}$ and $\eta_2 = \dot{\hat{x}}$

then

$$\dot{\eta}_1 = \eta_2 \qquad \text{(A7.5)}$$

$$\dot{\eta}_2 = g_2\eta_1 - g_1\eta_2 + t_o u_s \qquad \text{(A7.6)}$$

and

$$y_s = (\alpha_2 - g_2)\ \eta_1 + (\alpha_1 - g_1)\ \eta_2 + t_o u_s \qquad \text{(A7.7)}$$

Eqns. (A7.5), (A7.6) and (A7.7) are written as,

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g_2 & -g_1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ t_o \end{bmatrix} u_s \qquad \text{(A7.8)}$$

$$y_s = \begin{bmatrix} \frac{t_2}{t_o} - g_2 & \frac{t_1}{t_o} - g_1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + t_o u_s \qquad \text{(A7.9)}$$

Let

$$\underline{x}_s = [\eta_1 \quad \eta_2]^t$$

$$A_s = \begin{bmatrix} 0 & 1 \\ -g_2 & -g_1 \end{bmatrix} \qquad b_s = \begin{bmatrix} 0 \\ t_o \end{bmatrix}$$

$$C_s = \begin{bmatrix} \frac{t_2}{t_o} - g_2 & \frac{t_1}{t_o} - g_1 \end{bmatrix} \qquad d_s = t_o$$

Eqns. (A7.8) and (A7.9) are written as

$$\dot{\underline{x}}_s = A_s \underline{x}_s + \underline{b}_s u_s \qquad \text{(A7.10)}$$

$$y_s = C_s \underline{x}_s + d_s u_s \qquad \text{(A7.11)}$$